The Four 4s Challenge: A Narrative Inquiry into the Mathematical Curriculum Making of a Child

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Abstract: This article is based on research around the experiences of Chen, a Grade 2 student that shows his lived curriculum in the context of his school and home. This narrative inquiry is grounded in curriculum making and includes field texts in the form of Stavros's notes from our in-person sessions (prior to the covid pandemic), video recordings of Microsoft Teams conversations (during the pandemic), and some of his solutions to the Four 4s Challenge. For this paper, we illustrate the experiences coming to meet Chen, and the complexities and tensions around his lived curriculum as a strong mathematics student. Our intention is to provide insight into student-centred curriculum making.

Résumé: Cet article est basé sur une recherche autour des expériences de Chen, un élève de 2e année, qui montre comment il met en pratique le curriculum dans le contexte de son école et de chez lui. Cette enquête narrative est fondée sur l'élaboration des programmes et comprend des textes de terrain sous la forme de notes de Stavros issues de nos sessions en personne (avant la pandémie), d'enregistrements vidéo de conversations Microsoft Teams (pendant la pandémie) et de certaines de ses solutions du défi des 4. Pour cet article, nous illustrons les expériences vécues lors de la rencontre avec Chen, ainsi que les complexités et les tensions autour de ses expériences en tant qu'étudiant fort en mathématiques. Notre intention est de fournir un aperçu de l'élaboration de programmes d'études centrés sur l'étudiant.

Introduction

Chen [his given pseudonym] *is very excited to meet another person who loves math as much as he does! He already knows more than*

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most of us [the elementary teachers in his school]*, so we're very grateful that you've agreed to work with him. He's surpassed every assessment we've given him. He's also a human calculator. I asked him to multiply 34 and 56, and he gave the answer 1,904 in less than 5 seconds. We know he's bored in his math class because it's too easy for him, so we hope you can show him something exciting and help us meet his unique needs.* (Field text, May 2019).

Greetings. My name is Stavros Stavrou, and I work and do research in the areas of mathematics and teacher education at the University of Saskatchewan on Treaty six territory. One part of my job takes me outside the Department of Mathematics and Statistics to K-12 schools, where I work alongside teachers and students in their mathematics classroom. My focus is primarily working in schools with high Indigenous teacher-student populations. I accepted the research project with Chen on top of my regular duties so that I was not sacrificing time or financial resources in my existing programming.

Greetings. My name is Shaun Murphy and I work in the College of Education at the University of Saskatchewan on Treaty six territory. I was a long-time primary school teacher and then became a professor teaching elementary mathematics methods. I came alongside Stavros to intersect his academic mathematical knowledge with my experiences as an academic in teacher education.

Our past experiences working with children/youth and our current experiences with Chen are grounded in Connelly and Clandinin's (1988) work on curriculum making. One meaning of the term 'curriculum' is the mandated document that teachers consult when planning lessons. However, Aoki (1993) invited inquirers like us to think about the varied 'lived curricula' that is co-composed during the interactions between students and teachers. Working alongside Chen reminded us of how limiting mandated curriculum can be, as it did not serve his educative experiences. It was through Chen's excitement and playfulness doing mathematics that we knew the direction he wanted to go. Centring curriculum making shaped how we positioned our research alongside young students like Chen.

Locating Ourselves in the Broader Research

Mathematics teachers are tasked with finding innovative approaches to problem solving, reasoning, and contextualization for their diverse student audience. The principal of equity asks teachers to provide resources and pedagogical strategies that accommodate and support the varied needs of their students (NCTM, 2000).

Freiman (2011) summarized some of the issues surrounding differentiated mathematics instructions in Canada, noting that there are disparities in what can be offered provincially as there is no standardized way of approaching gifted children. Consequently, the pressure is on teachers to determine the advanced levels of their gifted students, and then develop suitable mathematical content and differentiated teaching methods. In our study, Chen's teachers provided the appropriate assessments to determine that his abilities were at a high school level—above what they would be able to support, they said. With Chen's parents' permission, we were contacted and asked to provide resources and opportunities to engage his learning. At our discretion, we were given freedom to determine an appropriate intervention, with one condition being that Chen still participated in his regularly scheduled class so that he could continue socializing with peers and learning the basics—no matter how easily he caught on. Therefore, our approach was to enrich his learning, rather than take an accelerative approach that might result in creating gaps in his foundational knowledge, mathematical language competencies, and social-emotional characteristics. (Heller, 2004; Hoogeveen et al., 2011; Sheffield, 2017).

Rotigel and Fello (2004) said some talented mathematics students demonstrate an "uneven pattern of mathematical understanding and development, since some are much stronger in concept development than they are in computation" (p. 47). Chen had an affinity to doing mental computations such as multiplying two- and three-digit numbers together, determining the prime factorization of three-digit composite numbers, and performing division to determine rates such as kilometres per hour. This was something he could do, but not something he found exciting. Rather, he was interested in the *how* and *why* of mathematical concepts.

Chen's teachers and parents said he often developed his own theories of number sense and other concepts using online videos at home, and he would ask them to explain these in more depth than they were able to. At the start of many sessions, Chen had a mental list of things his teachers could not answer, *so they told me to wait and ask you,* he said. It was clear immediately upon meeting Chen that he needed differentiated instruction and enrichment with advanced materials and curricula to reach his full potential (Rotigel, 2000; Tomlinson, 1995).

Admittedly, we had never encountered a child like Chen, but it was clear to us that his giftedness was a dynamic interplay of neuroscience, environment, and an affinity toward making patterns, experimentation, and logical reasoning. Sheffield (2017) provided a brief overview on some of the general meanings around giftedness in mathematics. She explained that some use inherent terms such as gifted, mathematically promising, talented, expert, and intellectually precocious to describe the aggregate of a child's abilities; while others—including us—prefer to focus on performance and development, rather than inherent qualities. Indeed, while we use accolades such as gifted, we see our interactions with Chen as a responsibility to further his development by attending to his unique needs. We did not analyze or code his thinking process, other than instances where he was asked to explain how he solved a problem. Our focus and philosophy was to present Chen with experiences that would excite him, appropriately challenge him, and inspire his creative disposition (Boaler, 2015; Feldhusen & Kroll, 1991; Sheffield, 2017).

That question is too easy and kind of boring. I can do that already, and I found another way to figure it out. Maybe think of something harder, a bored Chen would say. He craved challenging work, contrary to the responses we were accustomed to seeing from students (Stanley, 1991). Specifically, he enjoyed problems in which he could explore patterns and find multiple strategy solutions (Levenson, 2022), which led us to presenting him with the Four 4s Challenge that we will discuss later.

Our role as teachers is to provide problems requiring suitable levels of reasoning and lateral thinking, and to negotiate inquiry-based activities that give students some agency in developing metacognitive skills (Lester et al., 1989). Bouta and Paraskeva (2012) and Davis (1997) explained that teachers can practice cognitive apprenticeship through extrinsic support (via scaffolding, demonstrating, and mentoring) and intrinsic support (via scaffolding how learners construct meaning through reflection and interpretive listening.) Our experiences with Chen aligned with the roles that teachers are called upon to respond to capability and supportive learning opportunities (Diezmann et al., 2000).

Koshy and Casey (2005) used Sheffield's 3-dimensional model of breadth, depth/complexity, and temporal rate as a framework for ensuring teachers are attuned to students' mathematical progress. We never made an explicit connection to this model, but it was something we intuitively understood from our experiences teaching mathematics to a heterogenous audience of learners. Applying this to Chen was no different, and we were honored to learn alongside him.

Narrative Inquiry as Methodology

We used narrative inquiry as our methodology. Narrative inquiry is two pronged in its use of narrative (phenomenology) in that it inquiries into narratives and does so narratively (analysis). It is the study of experience, and in this research the experience of Chen and Stavros working together was captured narratively and then inquired into narratively. Connelly and Clandinin (2006) wrote about narrative terms for inquiry—referring to them as narrative commonplaces—which involved attention to place, sociality, and temporality. Connelly and Clandinin (2000) also wrote about the backward and forward, and inward and outward dimensions of narrative inquiry. In our work with Chen, we considered the backward understanding as his previous experience with school mathematics and his inward feelings toward school mathematics (and possibly mathematics in general) which shifted in his work with Stavros. Clandinin and Connelly (1992) wrote about the intersections of the narrative commonplaces alongside the backward/forward and inward/outward dimensions. This guided our considerations of the intersection of place and inward, by way of Chen's experience with mathematics and school, and then his work with Stavros at home and in a separate space at school. The shift between Chen's classroom and a separate resource room in his school seemed to be a normalized routine by the time Stavros began working with him.

We are cognizant of these intersections in regard to Stavros: his internal thoughts about mathematics, and his experiences across time as a student and then instructor of mathematics in various places. Narrative inquiry is a relational methodology and therefore we are aware that this space is shaped by both Chen and Stavros. A major tenet of narrative inquiry is that,

> People shape their daily lives by stories of who they and others are and as they interpret their past in

> terms of these stories. Story, in the current idiom, is a portal through which a person enters the world and by which their experience of the world is interpreted and made personally meaningful. Narrative inquiry, the study of experience as story, then, is first and foremost a way of thinking about experience. Narrative inquiry as a methodology entails a view of the phenomenon [...] To use narrative inquiry methodology is to adopt a particular view of experience as phenomenon under study. (Connelly & Clandinin, 2006, p. 477)

Chen shaped his daily life around his stories of experience (those he tells, and those others tell about him or to him). In this research, Stavros was able to capture Chen's experience narratively, recounting their time together and providing the reader with a temporal understanding of past experiences. The relational quality of narrative inquiry means we are people in relationship who simultaneously consider relationship and the role of sociality in our experiences (Murphy & Huber, 2019). Chen and Stavros were very much in a research relationship as they worked through mathematical experiences, both shaping new stories to live by. Attending to experience is foundational in narrative inquiry. Clandinin and Connelly (2000) wrote,

> Narrative inquiry is a way of understanding experience. It is a collaboration between researcher and participants over time, in a place or series of places, and in social interaction with milieus. An inquirer enters this matrix in the midst and progresses in the same spirit, concluding the inquiry still in the midst of living and telling, reliving and retelling, the stories of the experiences that make up people's lives, both individual and social. (p. 20).

In this quote our attention is directed to the experiential and relational quality of narrative inquiry, both of which factored in the research Stavros conducted with Chen. This research happened in the midst of Chen's life because he had mathematical experiences prior to interactions with Stavros and continued to have them during pauses in the research (such as summer holidays between school years.) Chen's family is relocating to

another province, so Stavros will negotiate virtual sessions with Chen and his family.
It is important to point out the following,

> the regulative ideal for inquiry is not to generate an exclusively faithful representation of a reality independent of the knower. The regulative ideal for inquiry is to generate a new relation between a human being and her environment—her life, community, world—one that "makes possible a new way of dealing with them" (Dewey, 1981, p. 175). [...] In this pragmatic view of knowledge, our representations arise from experience and must return to that experience for their validation. (Clandinin & Murphy, 2009, p. 39)

Working with these ideas, Clandinin and Murphy (2009) argued that when narrative inquirers "begin with an ontology of experience grounded in Dewey's theory of experience" (p. 599), which draws attention to experience as relational, temporal, and continuous, they arrive at an experiential, relational "conception of how reality can be known" (p. 599). In this way, an "ontological commitment to the relational locates ethical relationships at the heart of narrative inquiry" (p. 600). Stavros and Chen had a commitment to the relational shaped by a love of mathematics. Chen's experience was central to this research, as you will see later in the accounts of their interactions.

Method: The Enactment of the Methodology in Context

Stavros was contacted by teachers from an urban elementary school located in a Western Canadian prairie province. Chen was 5 years old and in Grade 2. When he met with his teachers for the first time, they explained his unique situation. In particular, they shared classroom observations of his overachievement in math and other subject areas. They showed assessments (provided by the school board) that they conducted to see the levels of his understanding in the common core areas. We examined these but did not obtain copies. The teachers explained that advanced students are typically ahead by one or two grade levels. With Chen, however, they could not adequately determine the ceiling of his availabilities and stated that he can solve problems taught in Grade 8 and 9. For example, he could factor polynomial equations, apply trigonometry to solve real-world problems, and use formulas to answer probability questions.

Consequently, the teachers' level of content knowledge to support Chen was, by their admission, insufficient. His parents understood that his remarkable abilities required analysis by university-level educators. Moreover, they were worried that he would become bored and disinterested in school. Everyone agreed that his curation required enrichment that would bring excitement to his learning.

After a few more consultation with his teachers, research approval and consent from his parents and the school board were given. Stavros began his work with Chen in May of 2019. They met once a week in one of the school's resource rooms during Chen's math class until the end of Chen's school year in June.

The sessions that took place during this time consisted of a somewhat diagnostic approach at determining specifically the topics that brought Chen joy. Simultaneously, differentiation and scaffolding guided how activities were presented.

They took a break over the summer holidays, and then continued to meet weekly again in September. The one-on-one sessions occurred in a resource room in the school until March of 2020, when school shifted to remote learning due to the pandemic. They continued their sessions over Microsoft Teams (a video conferencing service).

Stavros composed multiple kinds of field texts: field notes of observations made during his sessions with Chen, video recordings of sessions held virtually via Microsoft Teams, and artifacts of Chen's work. During their one-on-one sessions, Chen solved problems that Stavros prepared ahead of time, but they also engaged in topics that emerged as Chen directed his attention to other problems that piqued his interest (such as problems he was given by his teachers or problems that he had learned at home from watching YouTube videos.)

Chen was given a notebook to use specifically for our sessions. On the first few pages he copied things written on the Whiteboard, such as properties of exponents, graphs of linear and quadratic functions, and solutions to a few linear equations. He eventually stopped bringing his notebook, saying, *I don't need it. I remember what I did if you ask me the questions again.*

The fragments of stories throughout this article were composed either from Chen's words, the solutions he provided to the problems posed by Stavros, or from Stavros's interpretation of

Chen's recorded "actions, doing, and happenings, all of which are narrative expressions" (Clandinin & Connelly, 2000, p. 79).

They recorded the sessions during the periods that sessions were held online through Microsoft Teams due to Covid-19 constraints. The purpose of recording the sessions was to summarize interesting discussions to share in this article. His teachers and parents did not ask to view the recordings because Stavros adequately summarized the important details of his overall progress and performance. There were no audio transcripts.

Member checks consisted of conversations with Chen's parents and teachers. Monthly throughout the school year, Stavros had telephone conversations with Chen's parents, providing them information about the content covered and Chen's responses during the sessions. Chen's parents relayed to us how excited Chen was to be doing, *fun math*.

They maintained regular communication with Chen's teachers regarding the topic they covered together in class. Since Chen almost always understood his classroom lessons and completed his work, they were able to focus on the enriched activities. Sometimes these activities built upon his classroom work, while other times they introduced higher-level open-ended activities. His teachers did not give feedback on what was covered during his sessions; mostly because many of the topics were above their level of content knowledge. Teachers gave autonomy and authority for Stavros to direct Chen's learning, and Stavros was largely motivated by the direction Chen's excitement and engagement took them.

Curriculum Making

Drawing on Dewey's (1938) theory of experience with attention to continuity, situation, and interaction, and Schwab's (1973) curriculum commonplaces of subject matter, learner, teacher, and milieu, Connelly and Clandinin (1988) understood curriculum as a life in the making and shaped an understanding of this interaction as curriculum making. Curriculum making can be understood with Clandinin and Connelly's (1992) work which suggested that curriculum "might be viewed as an account of teachers' and students' lives together in schools and classrooms" (Clandinin & Connelly, 1992, p. 392). This understanding of curriculum making they described as "in dynamic interaction" (p. 392). In this way Clandinin and Connelly provided "an image of the teacher as a curriculum maker" (p. 366); they also drew attention to the idea of curriculum not as mandated subject matter outcomes but "as a course of life" (p. 393).

Huber and Clandinin (2005) wrote "One of the places where lives meet in schools is in curriculum making" (p. 318). This understanding of the negotiation of a curriculum of lives, as children's and teachers' lives meet in classrooms, was grounded in the idea that "the composition of life identities, 'stories to live by' … [are] central in the process of curriculum making" (p. 318). As Chen's life met with teachers' lives in school around mathematics, a made curriculum of mathematics was enacted. In his work with Stavros a further curriculum of mathematics was made. Therefore. if we follow their thinking, teachers are curriculum makers and so are children, exemplified by the ways Stavros and Chen made a curriculum together.

Their curriculum making was centred around the subject matter of mathematics. Stavros had been contacted by the parents and began working with Chen to augment and challenge his mathematical knowledge in ways that are explained further in the paper. Stavros has a BSc and MSc in mathematics, which guided the topics that Chen enjoyed doing. We do wonder about the applicability of this work to school mathematics and believe that with proper classroom supports and scaffolding—as shown with Chen—more complex mathematical concepts are achievable by young children. Therefore, the two of them created curriculum together through the ways Stavros's teaching reflected an intimate understanding of what Chen was capable of in mathematics learning. This entails knowledge by a teacher of the learning ability and style of the learner. We use the word *teacher*, not *schoolteacher*, by its use that anyone who imparts knowledge is a teacher.

Wilkins et al. (2006) described characteristics of curriculum making in mathematics as integrating mathematics across disciplines, focusing on problem solving, and modifying creating activities. It is evident in the experiences we share in this article that Chen was introduced to terminology, symbols, and processes used in physics and computer science to solve problems. Guiding Chen toward practical computations and discerning what he requires to progress exemplified how mathematical communication is part of co-composing curriculum with students.

Curriculum making is not about prescribing exercises to solve; it involves negotiating a level of independence to explore content while providing students opportunities to develop attributes of perseverance, imagination, and curiosity (Piirto, 1998). Curriculum making is a dynamic and ongoing relationship that centred the life and opportunities of Chen (Kim, 2006).

Identity Making

Closely tied to the idea of curriculum making is *identity making* (Connelly & Clandinin, 1988), which is also tied to the term *stories to live by* (Connelly & Clandinin, 1999). When used with curriculum making and identity making, stories to live by implies the curriculum that is made along with the consequent identity making, which shapes the stories by which we live in the world—including stories we tell ourselves and others. Chen told a story of being adept at mathematics. He told himself this story and it was a story told about him by others—his parents and teachers—who engaged Stavros to work with him in mathematics education.

By examining Chen's narrative coherence (Okazaki et al., 2014), Stavros recursively developed and modified activities based on Chen's existing mathematical knowledge, affinity to acquiring concepts outside of school settings, and his engagement. In particular, Chen was interested in solving exercises using concepts not encountered before. Therefore, Stavros could transition between theory and skills that were not necessarily formulated or grounded in previous iterations. Consequently, his facility with mathematics became one narrative expression. This coherence led to his identity making around understanding complex mathematical concepts. For example, Stavros noted,

> I was curious to know how Chen would respond if I gave him questions encountered on scholastic aptitude tests (SATs). I went to various websites and we did practice exams together (for example: https://blog.prepscholar.com/complete-list-of-free-sat-math-practice). I thought to do this for two reasons: the first is that math components of SATs often assess different branches of mathematics (number theory, combinatorics, probability, algebra, geometry, trigonometry, and other topics from precalculus), so I wondered how he would respond to tests that have multiple choice answers, in particular, to see if he would use solutions and work backwards to solve the problem. The second reason is the cliché around pop-culture notions that over-achieving students excel at SAT and

> other entrance exams to 'prestigious' colleges. If he got stuck, it was usually due to terminology. Usually, if I defined a word, phrase, equation, or theorem (sometimes by way of an example), he was able to apply them almost immediately. (Field text, September 2022)

The field note captures the idea of the complexity of mathematical concepts that Stavros was able to challenge Chen with during their sessions. In this way, we illustrate Chen's identity making was a process he was involved with in relation to his work with Stavros. Clarke (2016) wrote,

> Increasingly, we are called upon to reconsider the privileging of 'identity as presence' and to displace it with the notion of 'identity as effect' . . . We are being asked to consider identity not so much as some*thing* already present, but rather as production, in the throes of being constituted as we live in place of difference. (p. 205)

Chen and Stavros's work together entailed the production of their identities around mathematics. During their first session together, Stavros realized near the end that he was speaking to second-grade Chen as if he was one of his first-year university students. For example, Stavros used the mathematical term *distributive property* to explain how multiplication interacts with addition, and wrote on the board:

$$x(y+z) = xy + xz$$

Chen repeated the term *distributive property* back to Stavros once and then remembered its meaning in subsequent sessions without any reminders or prompting. In fact, with sufficient examples, Chen was able to remember any new mathematical terms or processes he was taught (for example, *commutativity, binary operation, exponentiation, polynomial,* and *factorization,* to name a few.)

Chen never acted shy and told his teachers how excited he was to see Stavros at their next sessions. For this article, we selected pieces of the field texts that reveal Chen's lived curriculum.

The Four 4s Challenge

The following is part of a reflection Stavros had during a session with Chen back in May 2019. It is told through Stavros's first-person voice.

It was clear to me that Chen wasn't interested in the project I brought for our session one particular day. Sometimes it happened that way—Chen would not engage in what I was presenting. An example is presented in a September 2022 field note below.

Chen: Why is your chair different than the other ones? It has wheels and a soft seat.
Stavros: I'm not sure. Mine might be a chair for a teacher, and the others are for students.
Chen: Do you need a soft seat because you are old?
Stavros: [mouth drops] I'm only thirty…
Chen: You are almost five times older than me. Wow!
Stavros: Chen, how would you balance the equation I've written on the board?
Chen: My table wobbles when I lean on it, and it isn't the same size as the other ones.
Stavros: I'll put a folded piece of paper under the leg. There. It doesn't wobble anymore. So, how do you get x by itself?
Chen: Oh, that is super easy. Sometimes you give me such easy questions that it's boring. No offense.
Stavros: [In a joking tone] Ha! I'm more offended that you think I'm old…
Chen: Subtract 10 from both sides. Now x^2 *equals 36, a perfect square. So,* x *equals positive 6 and negative 6.*
Stavros: What numbers do you find not *boring?*
Chen: I like making big numbers. In my computer game at home, I could pick the numbers I want to use. Sometimes I choose numbers so big that the game doesn't work.

In a subsequent session, Chen told me the ways he chooses numbers and then performs operations—such as addition, subtraction, multiplication, division, and exponentiation—to create new numbers.

Stavros: What do you mean?
Chen: 1000 divided by 2 is 500. Then 500 plus 100 is 600. Then 600 times 3 is 1800, and 1800 subtract by 300 is 1500. Actually, I found a bunch of different ways to do it.

Chen inspired me to think of another activity that will allow him to perform processes he finds interesting. I asked him if he heard of The Four 4s Challenge. When he said no, I gave a very simplified explanation that the goal of the challenge is to use exactly four 4s in any way possible to create whole numbers. (The interpretations of this problem are varied—see Anderson (1987), Ball (1971), and Wheeler (2002).)

As an example, I wrote on the Whiteboard:

$$0 = 4 + 4 - 4 - 4.$$

Chen giggled and did not need any further explanation. He grabbed my red marker, stood on a chair (he was too short to access most of the Whiteboard) and wrote:

$1 = 4/4 + 4 - 4$
$2 = 4/4 + 4/4$
$3 = (4 + 4 + 4)/4$
$4 = 4 + (4 - 4)/4$
$5 = (4 \times 4 + 4)/4$
(Field note, May 2019)

There is a lot to appreciate from this field note. One remarkable point is his attention to representing the order of operations correctly, including his use of parentheses. Even at the university level, I have to emphasize the use of parentheses to my students. There were a few instances where Chen would accidentally use more than four 4s or when I asked him to clarify an ambiguous expression. Another remarkable point is the amount he already understood about doing this type of calculation. This speaks to his home milieu that reveals the content of mathematics he encounters while exploring at his own pace, moving towards processes and topics of interest to him.

Stavros understood Chen's affinity towards doing a sequence of calculations and therefore transitioned away from the planned session so that Chen could explore what captivated him. Chen's response to the challenge confirmed to Stavros the importance of drawing on experiences from home.

Results

We organized the results of our interactions through various field notes containing narratives of some of the sessions. This is organized into the following subsections: *Mathematical Playfulness*; *Relational Curriculum Making: Playfulness, Creativity, and Invention*; and *Exploring Branches of Mathematics*.

Mathematical Playfulness

At subsequent sessions, Stavros and Chen continued to work through the whole numbers with the goal of getting to 100. There were numbers Chen would solve quickly and others that required more pondering. For example, Chen was stuck on 19. Stavros learned to ask Chen 'Have you ever heard of …?' Rather than assuming what Chen might or might not already know. Chen often playfully giggled whenever Stavros introduced a concept that he found amusing.

Chen: I can't figure out how to get 19. Odd numbers are harder, I think.
Stavros: Are they? How come?
Chen: I just have a feeling about it. Maybe. I need to think about it more.
Stavros: I can suggest something that might help. Have you ever heard of a factorial?
Chen: [Giggling] No, what is a factorial? Factorial—that's a funny word!
Stavros: A factorial of a non-negative integer is the product of all positive integers less than or equal to it. We use an exclamation mark for the factorial symbol. For example, 4 factorial is written as:

4! = 4x3x2x1.

What does this equal, Chen?
Chen: 24.
Stavros: Good! So, let's expand our mathematical toolkit to include 4!. It uses a single 4 to get 24. Play around with that.

Chen giggled at the new concept, pondered for a moment, and then wrote:

$$19 = 4! - 4 - 4/4.$$

He continued excitedly and quickly wrote:

$$20 = 4! + 4 - 4 - 4,$$
$$21 = 4! - 4 + 4/4$$
$$22 = 4! + 4 - 4 - \sqrt{4}$$

Seeing the relationship between factorials and multiplication led Chen to wonder if there was a symbol that would involve addition so that he could represent 4 + 3 + 2 + 1 using one 4. I explained that the capital Greek letter sigma Σ is used to represent addition; but to be more precise and correct with notation, I suggested using s(4) (said as '*s of 4*') to mean 4 + 3 + 2 + 1.

Chen: Cool!

He put these new ideas together and asked if s(4!) = 300. I opened the calculator app on my phone (I'm admittedly not as fast at mental math as Chen if you can believe it!) and verified his answer.

Stavros: How did you calculate that so quickly?
Chen: I think s(4!) means s(24).
Stavros: Ha! Yes, but how did you add 24 + 23 + 22, and so on, so quickly?
Chen: I did 25 times 12.
Stavros: Can you explain this to me?
Chen: I did 1 + 24 is 25; 2 + 23 is 25; 3 + 22 is 25. Like that. Then 12 + 13 is 25. So, 25 times 12.

I was astonished that Chen used the process that leads to a proof of the famous summation formula:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

I never provided him with this general formula or its proof, and I know it is not something his teacher would have covered, so his application of this process was something he learned from his self-directed exploration. Realizing Chen's ability to see patterns that are used in inductive reasoning, I kept a mental note that I wanted to teach Chen techniques of proving after we finished this challenge.

As he progressed through the challenge, there were many instances where he would insist on finding other ways to write a certain integer. He asked questions along the way, such as 'Do you know the total amount of ways that we can make each number?', 'Are there some numbers that only have one way?', 'Does this work for negative integers?', and 'What's the highest number *you* ever did?'

At least for mathematicians, it is common for such questions to arise when solving a problem. These curiosities about finding a general or exhaustive solution are what drives the field. It also shows the ways mandated curriculum in classroom learning hinders educative experiences for children like Chen who require something different. Centring his lived curricula means allowing him to pull me in different directions and co-compose his learning. This led him to wonder about other mathematical tools he could 'play with'.

Relational Curriculum Making: Playfulness, Creativity, and Invention

As Chen progressed through the challenge, Stavros offered hints at points that he felt Chen needed another expression he could use in his 'toolkit'. For example, Chen had been using .4 (decimal 4, without the 0 in front, to be consistent with the rule of only 4s appearing) to solve some of the problems but encountered instances in which he either had the answer using three 4s (and thus he wanted a way to transform 4 into 0) or was 1 short of making a number (and thus he wanted a way to transform 4 into 1). Stavros suggested that Chen think about what he can do with .4, to which Chen replied that he wanted to round .4 to 0. Stavros explained that computer scientists often use a floor function that rounds a real number, n, down to the greatest integer less than or equal to itself. Stavros wrote: floor(.4)=0, and said 'the floor of decimal 4 equals 0.' Chen used this to write:

$$52 = 4! + 4! + 4 + \text{floor}(.4).$$

Chen assumed there must be a function that rounds up so that he could get 53 easily. Stavros introduced him to the ceiling function:

ceil(.4)=1, said 'the ceiling of decimal 4 equals 1.' Chen used this to write:

$$53 = 4! + 4! + 4 + \mathrm{ceil}(.4)$$

Stavros and Chen spent approximately nine one-hour sessions solving the Four 4s Challenge. There were days when he solved 10 to 15 numbers, and others where he managed only a few. He never gave up and he never asked to switch to something else. He met the challenge with so much excitement, and a wonder of what new concept Stavros would provide for his mathematical 'toolkit'. If Chen was stuck, he invented what he needed to solve the problem. Some of his inventions were existing mathematical ideas, others were completely novel and truly clever. He exemplified Georg Cantor's belief that the essence of mathematics lies in its freedom (Carey, 2005). One might think that solving larger numbers might be more challenging, but Chen found it easier as he developed new strategies and tools.

Chen used a calculator on his tablet and played around with different calculations to see if he could generate new values to use in the problem. For example, beginning with:

$$4/.4 = 10,$$

he next considered 4/.44 which equals $9.\overline{09}$. (Chen already encountered elsewhere using bar notation to represent repeating decimals that do not terminate, such as 9.090909… = $9.\overline{09}$.) He then tried 4/.444 to get $9.\overline{009}$, and 4/.4444 to get bar $9.\overline{0009}$. Chen realized the more 4s he added, the closer he got to 9. This led him to write $4/.\overline{4} = 9$, which he then used to get:

$$81 = (4/.\overline{4})^{4/\sqrt{4}}.$$

Watching Chen's creativity and playfulness led Stavros to using unconventional methods. For example, Stavros considered writing 44 in its expanded form 40 + 4 so that he could look at a way of manipulating the digits in the ones and tens place using the notation:

$$\mathrm{digits}(4,4) = 40 + 4.$$

Using three 4s, Chen applied this new idea to write:

$$\text{digits}(\sqrt{4} \text{ x } 4,4) = 80 + 4 = 84.$$

Chen found this very amusing and used it in a number of solutions, such as:

$$85 = \text{digits}(\sqrt{4} \text{ x } 4,4) + \text{ceil}(.4).$$

Chen often said he needed to try different things to create odd integers.

By the time Chen hit 90, he had collected such a vast toolkit that he quickly solved numbers 91 to 100 within thirty minutes. For example, using 4% (i.e. 0.04) as one of his tools, he determined:

$$92 = 4/4\% - 4 - 4.$$

Exploring Branches of Mathematics

Stavros used every opportunity to show Chen as many ideas as possible. The Four 4s Challenge was a segue to discussions about different branches of mathematics. For example, Chen encountered classes of functions while exploring videos at home, so Stavros built upon this and introduced the logarithmic function f(x) = ln(x) and exponential function g(x) = e^x using their graphs and a table of values. Chen had an understanding of functions as input-output machines, so he explored f(4), f(44), g(4), g(.4), etc. However, Chen felt that using e^x was not allowed because Euler's number e (approximately 2.71828…) was a number different from 4, which went against the rules.

During the challenge, Chen identified integers that were prime and often thought of the factorization of composite integers. Such ideas are considered in the branch of mathematics called number theory. Stavros introduced the Euler Phi function, φ (n), to be the number of positive integers less than or equal to n that are relatively prime to n. For example, φ (16) = 8 because there are eight positive integers between 1 and 16 that share no common factors with 16 (these are 1, 3, 5, 7, 9, 11, 13, and 15). To use this function according to the rules, Chen wrote $\varphi\,(4^{\sqrt{4}})$. This used two 4s to get 8. Chen found this function interesting but felt it did not help because he already had ways to get 8 using two 4s (for example, 4 + 4 or $\sqrt{4}$ x 4). Together, they calculated $\varphi\,(4^4)$=128 (meaning, there are 128 integers between 1 and 256 that share no common factors with 256.)

Reflections and Concluding Remarks

We do not propose that this study prescribes a specific method to advance a child's learning. It is not our intention to provide a replicable framework for which other educators can bring to their classrooms. Rather, we wanted to exemplify how we enriched Chen's learning. We do not presume to know how, for example, the Four 4s Challenge can logistically be taken up in other homes or heterogenous classrooms full of students with varying mathematical abilities and proclivities. We can, however, briefly summarize some of the more generalizable ways we came to identify Chen's needs, which we hope will provide insight into possible implications of other educators and parents who might encounter similar situations at home and school.

Stavros is among some members in the Department of Mathematics and Statistics at USask who do outreach work in the community, which includes supporting mathematics teachers in K – 12 classrooms. It is not uncommon for educators and local school boards to contact our department seeking our expertise and asking for resources, particularly in underserved communities. We take on additional work with the community on a case-by-case basis. Working with Chen has been a unique experience so far. For other educators reading this work, we encourage you to contact local post-secondary institutions to see how you can be supported. We do not claim Chen's teachers are not capable of enriching his education. Rather, we are illustrating ways our roles as university educators and researchers provide us flexibilities, access to resources, and advanced specialties his schoolteachers are not guaranteed.

We acknowledge there are complexities surrounding sustainably attending to Chen's unique needs. Readers of this research may wonder about the deservedness, political, financial, logistical, and moral implications of providing arguable levels of attention to a single student or students such as Chen. A nuanced critique of this is beyond the scope of our work. However, we can explain that to mitigate excessive attention and to foster his independence, self-directed learning, and resourcefulness as a learner, Chen's sessions included being introduced to open-source digital resources and interactive software (e.g., Desmos.) Ultimately, Chen is in a school system in which he will have access to advanced placement courses and apprenticeship programs as he gets older.

We were partially informed by Heller's (2004) framework for identifying gifted students. One point of the framework is to have a multidimensional understanding of giftedness, which guided us to focus on Chen's personality and the social conditions of his learning, rather than looking at how he would score on standardized assessments.

Another point of the framework is to consider the functions and benefits versus the dangers of identification measures. For example, we understand the purpose of using identifiers such as gifted and talented in literature and research. However, we did not use any identifiers around Chen. He is aware that he is stronger at math than his peers, but we chose to support and guide his learning with positive reinforcement that did not impose accolades.

The delivery of the sessions was often trial-and-error. Based on his engagement or lack thereof, we would switch tasks. We also became aware with the branches of mathematics he enjoyed and therefore were better able to specialize his activities. Knowing his orientation to calculations and discovering multiple solutions were one of the inspirations to the Four 4s Challenge.

It would be remiss not to mention that at the onset of their meetings, it was often a struggle finding activities that sincerely captivated him. The successes came from nurturing his childlike curiosity and playfulness. Indeed, Stavros attended to the necessity of exploring other areas of mathematics when considering approaches to problem-solving. Stavros's background doing mathematics research in algebra informed his confidence in the subject (Creecy, 2020; Strand & Mills, 2014), and consequently he was able to impart to Chen the importance of engaging with different mathematical areas.

As educators, we must re-imagine ourselves as curriculum makers by considering the lives of our students. While expectations on teachers exist, lives—not mandated curriculum—need to be the starting point. We further need to actively maintain creative conditions and constructivist activities that guide students through thinking processes and representing their mathematical knowledge (Mainali, 2021; Zain et al., 2012).

Chen's ability to conduct independent inquiry and actively participate is a striking result of the ways we co-compose and perform our mathematics sessions. As mathematics educators in K-12 and post-secondary spaces, we have seen many instances of a passive acquisition of knowledge, which is in stark contrast to

Chen's actively learning, due to the ways we were able to serve Chen's needs through a student-centred approach that engaged his life as a child.

In terms of identity making, Chen made explicit that he wanted to be able to do mathematics every day like Stavros does. Chen was curious about problems that Stavros researches, and began to see himself as a mathematician who also solves problems. Through the relational curriculum they made, Chen's identity took a new shape in which he saw himself as *mathematical*.

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