

# **Conceptual Models in Educational Research and Practice**

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In this paper we suggest that the use of conceptual models, as developed in cybernetics and used in cognitive psychology, may help to bridge the gap between educational research and teachers. The notion of model is discussed in the context of explanatory theories, teachers' understanding of students, and students' understanding of mathematics. Piaget's concept of accommodation is explicated in terms of his scheme theory. A plea is made for greater flexibility, both in research methodology and teaching, and for an increased awareness that understanding and motivation to understand require the students' deliberate autonomous operating.

Dans cet article les auteurs proposent que l'utilisation de modèles conceptuels tels que ceux développés en cybernétique puis appliqués en psychologie cognitive peut aider à réduire le fossé qui sépare les chercheurs en éducation et les enseignants. Comme dans la pensée de Piaget, le concept d'accommodation ne se comprend que dans sa relation à celui de schème, ainsi la notion de modèle pourrait servir de structure de base, intégrant et éclairant les théories explicatives, la compréhension qu'ont les enseignants de leurs étudiants, et celle qu'ont les étudiants des mathématiques. Les auteurs soulignent le besoin d'une plus grande flexibilité tant en recherche qu'en enseignement. Ils rappellent un fait expérience, à savoir que la compréhension et la motivation à vouloir comprendre suppose chez l'étudiant une action délibérée et autonome.

Traditionally there has been a certain amount of detachment between teachers of mathematics and cognitively oriented educational scientists who endeavored to develop theories about the learning of mathematics. At present, however, there are signs of a rapprochement, at least on the part of some of the scientists, who have come to realize that their theories must ultimately be evaluated according to how much they can contribute to the improvement of educational practice. Healthy though this realization is, it at once raises problems of its own. At the outset there is the research scientists' inherent fear of getting bogged down in so many practical considerations that it will no longer be possible to come up with a theory that may satisfy their minimum requirements of generality and

elegance. Then, when scientists do come up with a tentative theory, there is the difficulty of applying it in such a way that its practical usefulness is demonstrated. This would require either scientists' direct involvement in teaching or the professional teachers' willingness and freedom to become familiar with the theory and to incorporate it into actual teaching practice for a certain length of time. In both cases, it will help if scientists and teachers can establish a consensual domain. In other words, they must come to share some basic ideas on the process of education and the teaching of mathematics in particular.

The main objective of cognitively oriented educational scientists is to understand how children build up their picture of the world piece by piece. In mathematics education the specific objective is to understand the way children build up their mathematical reality and the operations by means of which they try to move within that reality. The scientific investigators who enter this still largely uncharted field cannot search for some ultimate truth but must strive to understand individual children's understanding. The way to do this is to embed oneself as best one can in the actual situations where children manifest their constructing and solving procedures. First-hand observation, not only of children's mathematical activity, but also of *how that activity can be influenced*, is necessary to gain the experiential basis for formulating explanations of it. The researcher must actually teach children (Cobb & Steffe, 1983).

The gap between classroom teachers and contemporary educational scientists can be bridged only if they both begin to speak the same language — the language that springs from the researchers' quest to understand children's understanding and the teachers' quest to facilitate children's understanding.

David Hawkins (1973) said in one of the main lectures presented at the Second International Congress of Mathematics Education:

The really interesting problems of education are hard to study. They are too long-term and too complex for the laboratory, and too diverse and non-linear for the comparative method. They require longitudinal study of individuals, with intervention a dependent variable, dependent upon close diagnostic observation. The investigator who can do that and will do it is, after all, rather like what I have called a teacher. So, the teacher himself is potentially the best researcher. (p. 135)

If Hawkins is right, mathematics teachers should reconceptualize their goals and activities so as to include at least the *idea* of experimentation. Hawkins was optimistic and seemed to believe that the teacher's necessary reconceptualization could be brought about "if only we could offer him strong intellectual support and respect his potentialities as a scientist" (p. 135).

While it may not be clear what will have to be done to enable teachers to participate in experimentation in their classrooms, it certainly seems essential to shift the focus of their attention away from what students ought to be doing to an

attempt to discover what actually goes on in students' heads. In the traditional view, school mathematics is considered as something that exists in its own right, a well-defined and well-described body of knowledge that is to be transported from teachers to children. This view of school mathematics has its roots in the belief that mathematics is the way it *is* rather than the way human beings make it. This belief has permeated mathematics education at all levels. The traditionalists essentially ignore the mathematical knowledge of children and proceed on the formalist's assumption that there is a world of mathematical objects, a world independent of the thinking individual (Brouwer, 1913). In a relativistic world view, various frames of reference must be taken into account. Taking children's frames of reference into consideration, as well as those of adults, has made us aware of how limited traditional educational practices are. Once one gives up the idea that knowledge is a commodity that can be transferred from a teacher to a learner, it becomes imperative to replace that idea with some notion as to what *does* and *could* go on in the learner's head.

In this paper we try to show that the notions of teaching and of scientific research can be modified and that considerable common ground can be discovered once it is realized that both activities are essentially involved with the building of hypothetical models — models of particular students' cognitive structures in the one case, and models of the cognitive (or mathematical) development of children in the other.

### *The Notion of Model*

As we are using it here, the term *model* is borrowed from contemporary philosophy of science which has brought about a major revolution in the way we see the world and the relation between the world and scientific knowledge. The source of this revolution lies in the realization that observers will consciously *see* only what they are able to conceptualize, given their present repertoire of concepts. This idea is certainly not new. Protagoras, in the 5th century B.C., said "man is the measure of all things"; Hanson, in the 20th century A.D. (1958), wrote: "There is a sense, then, in which seeing is a 'theory-laden' undertaking. Observation of *x* is shaped by prior knowledge of *x*" (p. 19). Both statements can be read to mean that one sees what, in a wide sense, one is ready to see.

This is different from the conventional 19th century view of the scientific endeavor characterized by Brush (1974), in which ideal professional scientists are seen "as rational, open-minded investigators, proceeding methodically, grounded incontrovertibly in the outcome of controlled experiments, and seeking objectively for 'truth'" (p. 1164).

In contrast to this, scientists who have founded and advanced their work on the revolutionary conceptual changes brought about by relativity, quantum mechanics, genetic epistemology, and cybernetics tend to be extremely wary of

the notions of truth and objectivity in the ontological sense. They have become very conscious of the fact that observation is a generative rather than a passive process, that all data are "theory-laden," and that, as von Foerster (1974) so neatly put it, "objectivity is the illusion that observation could take place without an observer" (p. 7).

Just as the interpretation of a piece of language is always guided by the individual interpreter's experience and expectations, so the interpretation of what one observes is always governed by some theory one has in mind and a goal one has chosen.

Lakatos (1970) addresses this point in his discussion of research programs. According to him, the *hard core* of a scientific research program consists of those beliefs which the researchers in the program do not challenge. The *protective belt* of the hard core (which is held to be irrefutable)

has to bear the brunt of tests and gets adjusted and re-adjusted, or even completely replaced, to defend the thus-hardened core (p. 133); . . . research policy, or order of research, is set out — in more or less detail — in the *positive heuristic* of the research programme . . . the positive heuristic consists of a partially articulated set of suggestions or hints on how to change, develop the 'refutable variants' of the research programme, how to modify, sophisticate, the 'refutable' protective belt.

The positive heuristic of the programme saves the scientist from becoming confused by the ocean of anomalies. The positive heuristic sets out a programme which lists a chain of ever more complicated *models* simulating reality; the scientist's attention is riveted on building his models following instructions which are laid down in the positive part of his programme. (p. 135)

It is the confirmations or verifications that sustain a scientific research program. The anomalies (i.e., the experiences that do not fit) are ignored but not forgotten, for it is hoped that they can eventually be turned into corroboration of future models which will supersede the current ones.

For Lakatos, theories were hypothetical-deductive systems and the theoretical activity of the scientist constituted the primary source of progress of a research program. From the constructivist perspective, one particular aspect of this position must be stressed. Even the hard core of a research program (i.e., those beliefs that the program takes for granted) should not be considered as eternal truth. The history of science shows that, no matter how successful research programs are, they eventually collapse and are superseded by others with different hard cores. In fact, the *knowledge* constituted by a hard core is viable as long as it serves the purposes and helps to attain, with the help of the models in its "protective belt," the goals of the established community of researchers. When it ceases to do this and, in Lakatos' words, "degenerates" — either because some anomalies can no longer be disregarded for practical reasons, or because the goals of the community are shifted — a new research program must

be generated.

A model, then, "simulates reality"; it is a conceptual construct that is treated as though it gave an accurate picture of the real world, but has the actual function of making experimental results and other experiential elements compatible with the general assumptions that are inherent in the research program's core.

### *Models in Educational Science*

The notion that learning can be modelled as a process of active self-organization has been developed independently in the fields of cognitive psychology and cybernetics (the discipline that investigates communication and control). From this perspective, the construction of models that "simulate the reality" of the learning organism becomes vastly more important than it was in the traditional learning theory where the learner was simply a passive entity whose responses were more or less mechanistically determined by external stimuli (Skinner, 1977). In retrospect it is indeed puzzling how radical behaviorists, either in psychology or in education, could live for so long with the peculiar contradictory notion that they, as experimenters or teachers, were free to make choices as to what their subjects should be doing, whereas the subjects were considered to be wholly stimulus-determined reactive entities.

If both educational scientists and mathematics teachers want to profit from this shift in the psychological paradigm, they must come to regard the learner as a relatively autonomous entity, an *intelligent* entity that is first and foremost concerned with making sense (i.e., some kind of order) in its own experiential world. This conviction forms the hard core of a new research program in truly *cognitive* psychology as well as the basis of a teaching methodology that is significantly different from the traditional one. In the traditional view, the child was a relatively malleable entity that could be shaped by good examples, by practice and drill, and above all by the judicious allocation of reinforcement. The really effective teachers, of course, have always known — at least since Socrates — that examples, drill, and overt reinforcement are quite effective in producing a desired behavior; but precisely because they were good teachers they also knew that generating *understanding* was a worthier educational objective than merely modifying behavior. To generate understanding is also a much more difficult objective to attain. To some extent, however, this greater difficulty is compensated by the fact that understanding is self-reinforcing because, from the children's point of view, it leads to *making sense* of their experiential world.

In adopting the new cognitive paradigm, then, it becomes imperative that both teachers and researchers acquire some theoretical notions of *how* this "making sense" can be conceptualized. The sum of these notions, once they are linked and in some way organized, constitutes what we want to call a *model* of the

cognitive or, better, the cognizing part of the child. But because the goals of the scientists and the teachers are not entirely the same, there will have to be several models, and though they must obviously all be related, they will be different in certain respects. First, there will be a general model of learning, a model that serves to illustrate and implement the main thrust of the cognitive core theory towards self-organization and the active construction of knowledge. Because this model is intended to be general and interdisciplinary, it will not take into account the particulars and difficulties that are inherent in teaching and learning mathematics. The construction of the particular concepts and operations of school mathematics would be the content of a second model that attempts to explain the successful learning of mathematics. It is at this level of specificity, where teaching and learning are quite deliberately restricted to the area of mathematics, that we as educators wish to foster interactions between researchers and teachers. To achieve this, however, a further distinction of relevant models is needed.

Both educational scientists and teachers in the field of mathematics education try to model the children's mathematical reality and how that reality may be cognitively built up piece by piece. The first, the scientists, may be mainly interested in establishing the hard core of a mathematics learning theory that would be applicable to as large a number of children as possible, but the viability of that theory, to quote Lakatos again, depends on confirmations of "verifications" that sustain a scientific research program. Consequently, in order to "confirm" or "verify" their theory, the scientists must "test" it by observing individuals. But — and this is crucial from the cognitive point of view — the tests in this context do not primarily concern the level of performance of new children but rather the question of whether or not the model can be maintained in the face of observations and teaching experiments with new children. However, it is not only in the context of justification but also in the context of reinvention that the scientific investigators need to observe individuals. In order to formulate even the most tentative model of cognitive change, educational scientists must witness the growth of mathematical knowledge in particular children and clarify and substantiate their interpretations by means of deliberate interventions. Conceptual analysis alone is simply not sufficient as a source of insight in model building. It is only on the basis of models of particular children that a more general model can eventually be abstracted — and the models of particular children are a natural bridge between educational scientists and the teachers.

Teachers may be predominantly interested in the progress of the individual children they are teaching; but their assessment of progress and, indeed, their very job as teachers would become practically impossible if their method of teaching mathematics had to be adapted in all details to each individual child. Teachers, therefore, need an at least partially generalized theory and a model of

the learner that is general enough to serve as a basis for the establishment of more than one individual model. Ideally, then, the teachers' models of individual students will be instantiations of the educational scientists' more general model of mathematics learning; and, conversely, the individual models the teachers construct for individual students will be a continuous testing ground for the theoretical assumptions the scientists have incorporated in the more general model.

Thus, both educational researchers and teachers, whether they like it or not, are in a very real sense dependent on one another. Given this dependence, the two roles also have much to offer each other in terms of assistance and exchange of ideas. As Hawkins (1973) pointed out:

The working perspective of a teacher allows him to make many observations of those acquisitions and transitions in intellectual development upon which the growth of mathematical knowledge depends. But such a teacher is of course not only an observer, he would indeed be less of an observer if he were not also a participant; one who, because of the way he shares in and contributes to the development, can earn the privilege of insight into its details and pathways. (p. 117)

To make explicit this relationship between scientists and teachers, as we see it, it is indispensable to adjust the conceptions of scientific theory and the practice of teaching to some of the present ideas in the philosophy of science and, in particular, to adopt the notion of hypothetical models as interface between a basic theory and the experiential reality in research and teaching. The notion of the hypothetical model, however, brings with it a characteristic difficulty.

#### *The Instrumental Character of Models*

Working with children is in many ways like working with foreigners with whom one has only fragments of a language in common. The situation is extreme when the work involves numbers and mathematical operations and aims at developing some insight into how individual children think about numbers and how they operate with them. Anyone who has seriously tried to investigate what actually goes on in children's heads when they are struggling to solve an addition or subtraction problem at the limit of their present capability will have realized that the children's mathematical world is indeed outlandish from the adult's point of view.

Yet, children who have not been totally alienated from the number game and have at least a modicum of motivation do not act randomly. They do proceed according to some method, even if that method would seem unorthodox to the experienced reckoner. To get an inkling of what that method might be, investigators cannot but use their own imaginations and try to conceive a reasonable path that might connect such manifestations of children's operating

as can be observed, with steps that could possibly lead to an answer to the given question. That is to say, no matter how hard investigators try to adapt their analyses to the "foreign" ways of children, the model they build up will always be a model constructed out of concepts that are necessarily the investigators'. Because children's ways of thinking are never directly accessible, the investigator's model can never be compared to a child's thought in order to determine whether there is or is not a perfect match. The most one can hope for is that the model *fits* whatever observations one has made and, more importantly, that it remains viable in the face of new observations.

What, one might ask, is the use of such models if they remain hypothetical and are linked to the reality of children's thinking, not by hard facts, but by inferences that may be countermanded at any moment? The answer is simple and perhaps disconcerting: No one can ever discover hard facts about the thoughts or operations that go on in another person's head. The closest one can come is through linguistic communication, but even there one deals with approximation. Language does not transport pieces of one person's reality into another's — it merely prods and prompts the other to build up conceptual structures that, to this other, seem compatible with the words and actions the speaker or writer has used (von Glaserfeld, 1983).

There is no denying the uncertainty inherent in all conjectures about another's mental states and processes. Yet, it would be foolish to say that, because their accuracy is inherently uncertain, such conjectures should be considered useless. This inherent uncertainty pertains not only to psychology and its investigations of the mind but also to the "hardest" of the sciences (Popper, 1963). In this respect, then, a cognitively oriented educational methodology differs radically from behavioristically oriented ones. If the educator's objective is the generation of certain more or less specific behaviors in the student, the educator sees no need to ask what, if anything, might be going on in the student's head. Whenever the student can be made to produce the desired behaviors in the situations with which they have been associated, the learning process will be deemed successful. Students do not have to *see* why the particular actions lead to a result that is considered "correct"; what matters is that they produce such a result. From our point of view, this exclusive focus on performance differentiates what we would call *training* from the kind of teaching that aims at *understanding* (von Glaserfeld, 1989).

In contrast, cognitively oriented educators will not be primarily interested in observable results, but rather in what students *think* they are doing and *why* they believe that their way of operating will lead to a solution. The rationale of this shift of focus is simply this: If one wants to generate understanding, the reasons why a student operates in a certain way are far more indicative of the student's stage of conceptual development than whether or not these operations lead to a result that the teacher finds acceptable. Only when teachers have some notion of

the conceptual structures with which students operate, can they try to intervene in ways that might lead students to change something in these conceptual structures.

When such notions regarding states and operations in another person's head come to be organized in a coherent structure, they constitute hypothetical models. Though the states and operations that concern educational scientists are essentially the same that concern teachers, they will be organized somewhat differently because the goals of the two disciplines are not quite the same. The researchers want to build up a model that illustrates a way in which students construct mathematical knowledge. Because it would be much more useful to have one model rather than several different ones, researchers must focus on features which, they believe, will be widely generalizable. Above all, they must establish patterns of change and development, and basic principles of *how* change and development may be induced. Teachers, on the other hand, are primarily concerned with the progress of groups of students; therefore, they need first of all a plausible model of the conceptual structures with which students are operating at the time.

Ideally, then, the model or models teachers come to use in interactions with students should always be somewhat simplified or modified versions of the general model the educational scientists have developed for mathematics education. Insofar as teachers' models of their students must contain mathematics, this has to be the particular student's mathematics, and it is the teachers' business to infer as much about this as possible in their interactions with the students. These inferences, however, can then be fitted into the more general model of conceptual construction and change that has been developed by cognitive psychology and educational research. Clearly, if a generalized model of mathematics learning is to be grounded in practice, it must have been abstracted from a great deal of experience with actual students' successful learning efforts and actual teachers' successes in instruction. We present a preliminary outline of such a model in the next section. In any case, however, we believe that the fundamental relation between the researchers' and the teachers' models is what Hawkins alluded to in the passage quoted at the outset.

#### *Methodological Considerations*

One recent development based on the close relationship between teaching and research is the technique that has become known as the "teaching experiment" (Steffe, 1983; Cobb & Steffe, 1983). Designed to provide the experimenter with the opportunity to explore conceptual change and development in the learner, the technique is indeed a powerful tool for the construction of conceptual models. What has sometimes been misunderstood or ignored is that teaching experiments were originally designed not for the immediate purpose of teaching, but rather to serve the experimenter in the quest for more reliable models of the

learner. Although it does happen that a task, question, or suggestion presented within a teaching experiment benefits the student because it leads to a step in a new direction, any such direct benefit to the learner is not the main objective of the experimenter. Experimenters are excited whenever a student embarks on a new and successful step, but their interest is focused not on advancing an individual student, but on discovering something about the specific conditions under which students can be expected to take new steps.

The construction of mathematics learning models (which, with regard to abstraction, lie between the most general learning model and the models of individual learners) still relies on concepts in the hard core of the research program, concepts that transcend any particular model and have been accepted by the researchers (Steffe, 1984). These hard core concepts are used in the formulation of both the model builder's abstractions from experience and the model of the learner.

Thom (1973) stressed that, in order to raise to the level of conscious awareness of those cognitive structures that are only implicit in what children do, it is necessary to lead the children into situations where their results of using those structures can be experienced. In principle, we agree. But, as constructivists, we would insist that students can become aware of the kind of abstractions that are involved in mathematics only if they are already carrying out the operations from which the new conceptual structures are to be derived. (We do not see how anyone could make abstractions from anything except his or her own experiences and actions.) The main purpose of the teaching experiment is, in fact, to discover the kinds of situations that may induce a naive learner first, to embark on a novel way of operating and then, if this new way led to success, to abstract from it an operative conceptual structure.

Thom's (1973) notion of extracting conscious structures from unconscious activity is for us closely linked to Piaget's notion of accommodation, a notion which, unfortunately, is not as simple as many Piaget interpreters seem to believe. Thus, we frequently read textbook definitions that are essentially similar to the following: "While assimilation involved changing incoming information, accommodation involved changing the structures used to assimilate information" (Brainerd, 1978, p. 24). To anyone who reads the Genevan literature in the original language, this would at once be suspect because Piaget does not use the term "information." But let us focus on accommodation.

One of the most important aspects of a viable model of the learner would be an indication of at least some of the ways in which a learner's conceptual structures are superseded by structures that seem more adequate from the teacher's point of view. Such modification or increase in an individual's conceptual repertoire is indeed what Piaget called accommodation; yet, in his constructivist theory, change or growth does not take place in response to external information but in response to an internal perturbation.

The sort of perturbations that lead to the accommodation of conceptual structures are perturbations that arise in the context of a *scheme*. A scheme is a triadic arrangement consisting of an initiating situation (which may be perceptual or conceptual), an activity or operation, and an expected outcome or result (von Glaserfeld, 1979). According to the original scheme theory, there are three main causes of perturbation:

1. A situation is recognized as instantiation of one that has been associated with a particular action, the action is carried out, and the expected result fails to be experienced. (Note that the *recognition* of the trigger situation is where *assimilation* plays its part.) This first kind of perturbation, that is, the failure to produce the expected result, will most frequently lead to an accommodation of the recognition procedure.
2. If the scheme, as described in (1), produces, instead of the accustomed result, another result that turns out to be desirable, this may lead to differentiation in the initial situation or in the activity and thus to the construction of a new scheme which will then be expected to produce the new result.
3. A different activity, associated with a different situation, leads to a result that is recognized as the expected result of another scheme.

This, obviously, is an extremely general model of the learning situation and it should be clear that, in any area of experience that provides complex goals, the learner will soon become susceptible to other perturbations. Above all, with increasing familiarity and practice, the final component of the scheme, the *expected outcome*, tends to become a more diversified factor and its nonattainment will then lead to distinguishable subcategories of the perturbation listed under (1) above. The work of mapping and sorting out these ramifications is still very much in progress.

This work has, of course, its own problems. One of them is that it does not conform to the traditional procedure of experimental psychology, in that its progress is necessarily *ad hoc*. Educational researchers, in their role as observers in a teaching experiment, may at a given point decide to generate a perturbation on the spontaneous conjecture that the child in the experiment might be able to make a certain accommodation. Similarly, they may conjecture that the child made an accommodation. But there is no way to tell whether or not this is actually the case. The only way to confirm such conjectures is to engage the child in further interactions. However, what form these interactions should take with a particular child at a particular point in that child's conceptual development is not something that can be decided beforehand in any detail. It must depend on the experimenter's experienced intuition at the particular moment.

This flexibility, though it has at times been interpreted as a lack of methodological rigor, is absolutely essential if, for the purpose of devising a general model, we want to chart the growth of understanding in human learners. As we said at the outset, this undertaking is very different from the behaviorist's attempt directly to modify behavior. Understanding and the urge to make sense of one's experiences are the most private of affairs. But to become conscious of understanding and making sense are the very results of intelligent behavior that provide satisfactions to the acting individual and thus may generate the motivation to continue the process of learning. And if, as teachers, we want to foster understanding, we will have a better chance of success once we have more reliable models of students' conceptual structures, because it is precisely those structures upon which we hope to have some effect.

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