The Igloo and the Natural Bridge as Ultimate Structures

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ABSTRACT. The Eskimo snow igloo is not a hemisphere as frequently depicted, but a catenoid of revolution with an optimum height-to-diameter ratio. This shape eliminates ring tension and shell moments and therefore prevents failure by caving or bulging. Rainbow Natural Bridge, Utah, is a catenary, probably because of weathering along the trajectory of maximum compressive stress.

RÉSUMÉ. *L'iglou et le pont naturel comme structures idéales.* L'iglou de neige esquimau n'est pas un hémisphère comme on le décrit fréquemment, mais bien un alysséide de révolution dont le rapport hauteur/diamètre est optimal. Cette forme élimine les tensions annulaires et les moments de coque et prévient ainsi la rupture par enfoncement ou par saillie. Le pont naturel de Rainbow en Utah est un arc caténaire, probablement par suite de l'altération le long de la trajectoire de l'effort compressif maximal.

РЕЗЮМЕ. *Иглу и Естественный Мост как предельные структуры.* Снеговой иглу эскимосов представляет собой не полусферу, как это часто изображают, а катеноид вращения с оптимальным отношением веса к диаметру. Такая форма исключает кольцевое натяжение и радиальный момент и, следовательно, предотвращает обрушение путём оседания или вспучивания. Естественный Радужный Мост (Утах) имеет форму цепной линии, возможно, из-за эррозии вдоль направления максимального сжатия.

INTRODUCTION

An opportunity for a civil engineer to observe construction of an Eskimo snow igloo was afforded during the 1970 test voyage of the ice-breaking tanker, S. S. *Manhattan.* While the ship was stopped for ice tests near the north coast of Baffin Island, three Eskimos from the nearby village of Pond Inlet came to visit and demonstrate their igloo-building expertise (see cover picture). The igloo afforded an unusual opportunity to observe shape, as it was built entirely above snow level. This is in contrast to localities with more snow, where blocks may be cut from within the structure, lowering the floor level and reducing the outer dimensions of the igloo (Peter Noblet, personal communication 1972). Meanwhile the author performed shear strength and density tests on the same snow.

ARCHES AND DOMES

Modern and innovative uses of thin-shelled concrete structures give the impression that shells are a modern invention, but simpler masonry shells such as domes, cones, etc., ornament much architectural history and date from prehistory.

Nearly all architectural domes of civilization employ segments of circular or spherical arches, yet engineers and architects have known for centuries that an

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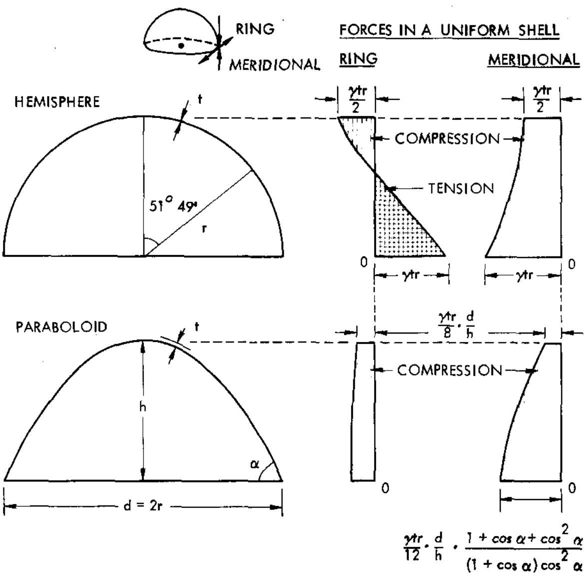

FIG. 1 Theoretical forces in two thin shells of revolution. Modified after Fischer (1960).

unreinforced masonry dome built in a spherical shape will collapse. The reason for the collapse is tensile ring stresses around the lower part of the dome (Fig. 1). Thus one of the "cold facts" about igloos is that constructed in the shape of a hemisphere, they should bulge at the base until the top falls in. Nevertheless the domes topping off major edifices of civilization from Roman times to the twentieth century are mostly spherical, either using only the upper cap of the sphere where the stresses remain compressive, or employing a variety of circumferential iron hoops, chains or masonry buttresses to hold the lower sides in. In contrast the Canadian Central Eskimos have evolved a mathematically more complicated design that is approximately a parabola in cross section, and more precisely a catenary. The catenary, from the Latin for chain, is the shape assumed by a chain held only at the ends. A modern example of a catenary in compression is the St. Louis Arch, Missouri, U.S.A. The equation for a catenary is derived in textbooks in engineering mechanics, and may be written $y = a(\cosh x/a - 1)$ where y is the height to any point in the surface, x is the horizontal distance to the same point, and a is a constant.

The stresses in an inverted paraboloid or catenoid are exclusively compressive; the latter has the additional advantage of zero bending moment everywhere within the shell. Thus as the snow in a catenoid igloo ages and undergoes compressive creep, the sides should not buckle.

If the walls are of uniform thickness and density, the maximum compressive stress at the base of a paraboloid is (Fischer 1960)

$$S_\alpha = \frac{\gamma d^2}{24\ h} \cdot \frac{1 + \cos\alpha + \cos^2\alpha}{(1 + \cos\alpha)\cos^2\alpha} \qquad (1)$$

where d is the diameter at the base, h the height, γ the unit weight of the snow, and $\alpha = \arctan 4h/d$. Since stress is a force per unit area, if the walls are of uniform thickness the compressive stress is independent of wall thickness —

thicker walls provide better insulation but do not strengthen the structure because of added weight. Thinner walls may be preferred to prevent melting of the interior surface (Jumikis 1966, p. 56).

Since α in equation (1) depends on the height-to-diameter ratio h/d, the terms may be rearranged and a stress term $S_\alpha \gamma d$ plotted as a function of h/d:

$$\frac{S_\alpha}{\gamma d} = f\left(\frac{h}{d}\right) \quad (2)$$

A plot of this function in Fig. 2 indicates that the compressive stress will be minimized if h/d = 0.3. However, since igloos gradually become shorter with time due to compressive creep of the snow, a larger h/d should be used to avoid loss of stability from an eventual upsurge of compressive stress brought about by shortening. The igloo on the cover of this issue of *Arctic* had h/d = 0.6, which is well above the critical value of 0.3 while increasing the stress term only from 0.42 to 0.52.

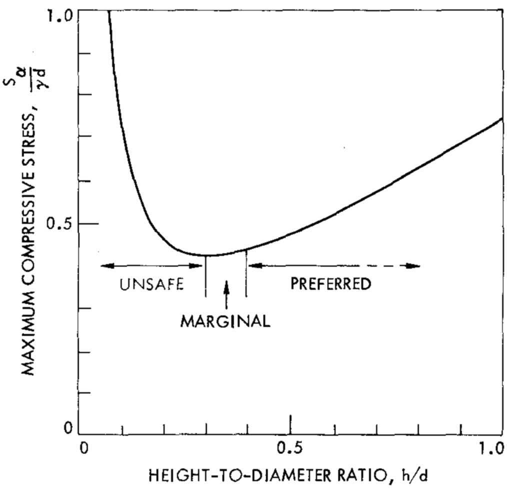

FIG. 2 Theoretical maximum compressive stress at the base of a uniform paraboloid as a function of height-to-diameter ratio.

The maximum compressive stress at the base of the igloo can be obtained from equation (2) and Fig. 2 by multiplying $S_\alpha/\gamma d$ times the snow unit weight γ and the mean igloo base diameter. For the igloo in the cover picture, the snow unit weight γ measured from the cut blocks averaged 0.43 gm./cc. Thus the maximum stress in the snow is

$$S_\alpha = 0.52 \times 0.43 \times 250 = 56 \text{ gm/cm}^2 \text{ (0.80 psi)}$$

IGLOO STRENGTH

The shearing strength of the igloo snow was measured by an instrument devised for testing soils, called the Iowa bore-hole shear test (Handy and Fox 1967) or sometimes the "Handy Hole-Puller". In this test a 7.6 cm.(3 in.)-diameter hole is bored, a grooved expansion head inserted and expanded against the sides of the hole at a predetermined normal stress, and the expanded head

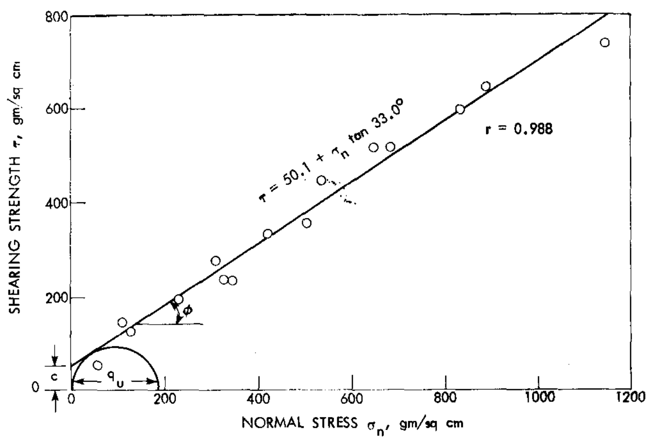

FIG. 3 Results from four bore-hole shear tests in the igloo snow, temperature = 10C.

pulled to cause shearing in the snow parallel to the axis of the hole. The pulling force to cause failure is measured, and the test is repeated at several stresses to develop a failure envelope. If the envelope is linear, it is described by a cohesion intercept c and an angle of internal friction ϕ. Advantages include speed, precision, and testing within the sample environment.

Pooled results of 4 bore-hole shear tests (equivalent to 15 direct shear tests), Fig. 3, indicate an average cohesion of about 50 gm./sq.cm. (0.71 psi) and a friction angle ϕ of 33 degrees. The unconfined compressive strength q_u may be derived from these data and from properties of a Mohr circle shown in Fig. 3:

$$q_u = \frac{2\ c \cos \phi}{1 - \sin \phi} = 185 \text{ gm./sq.cm. (2.6 psi).}$$

This is low compared to other published data (Nakaya and Kuroiwa 1970) but still higher than the stress exerted at the base of the igloo so that the engineering factor of safety, or strength divided by maximum stress, is 185/56 = 3.3, which is a rather good value for mathematically-engineered construction. A marked sensitivity of the snow strength to incipient melting and refreezing was noted: The first test, which was with an initially warm instrument, gave c = 335 gm./sq.cm., ϕ = 35.7°, and q_u = 1300 gm./sq.cm.

One may calculate the maximum safe-size igloo possible with a particular snow by substituting the unconfined compressive strength for S_α in equation (2) and using the ratio in Fig. 2 corresponding to h/d = 0.35. With S_α = 185 gm./sq.cm., γ = 0.43 gm./cu.cm., and a factor of safety of 1.2 to allow for imperfect fit of the blocks,

$$d_{max} = \frac{185}{1.2 \times 0.43 \times 0.35} = 1024 \text{ cm. (34 ft.)}$$

and

$$h = 0.35 \times 1024 = 358 \text{ cm. (12 ft.)}$$

In the case of an igloo cut into the snow, this height would be measured in the interior.

The igloo in Fig. 1 is characteristic of an overnight or hunter's camp. Boas (1888) reported that winter quarters igloos ran 360 to 450 cm. (12 to 15 ft.) in diameter. For larger accommodations several igloos were constructed and connected by vaults, suggesting a size limitation. Stefansson (1927) became adept at igloo-building and reported that the largest he had ever seen was a community "clubhouse" 900 cm. (30 ft.) in diameter, with $h/d = 0.35$, remarkably close to the theoretical optimum shown in Fig. 2. Stefansson used $h/d = 0.67$ for his own igloos and did not comment on the shape except to note it is nearly an "ideal dome". Perhaps the most detailed published instructions for building igloos are by Rowley (1938), who sketched them as nonspherical but did not emphasize that such a shape is essential.

The Eskimo igloo thus embraces a structural perfection arrived at by trial-and-error, without benefit or prejudice from mathematical theory. The design process constitutes an evolutionary optimization for design of domed masonry structures, matched but hardly surpassed by modern scientific engineering.

NATURAL BRIDGES

The catenary shape also appears to have evolved in Rainbow Natural Bridge, Utah, U.S.A. This arch was described and discussed by Livingston (1961) as an ellipse with an axial ratio which precludes tension at the top. Although the arch can be described by an ellipse with an axial ratio of 3.6, it is described equally well by a catenary, which is less general since it has one constant instead of two. Perhaps nature dislikes tension almost as much as it abhors a vacuum, and microfractures held open by tensile stress become localized sites for weathering with attendant expansion and spalling parallel to the direction of tension (Lutton 1969). Furthermore where tension does not exist as a boundary condition, tensile crack propagation still occurs within a brittle material normal to the direction of compression (Fairhurst and Cook 1966); thus the shape of a weathered natural arch in uniform nonlayered material should describe a trajectory of maximum compressive stress near the surface, i.e., a catenary.

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