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Affect, Motivation, and Engagement in the Context of Mathematics Education: Testing a Dynamic Model of Their Interactive Relationships[1]

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The present analysis aimed to test the dynamic (interactive) model of affect, motivation, and engagement (Linnenbrink, 2007) in mathematics education with a nationally representative sample. Self-efficacy, self-concept, and mathematics anxiety were indicators of pleasant and unpleasant affect. Intrinsic and extrinsic motivation were indicators of mastery and performance approach. Educational persistence and cognitive activation were indicators of behavioral and cognitive engagement. The 2012 Programme for International Student Assessment (PISA) supplied a sample of 4,978 students from the United States for structural equation modeling. The results indicated that the PISA data overall supported the dynamic model. Specifically, the PISA data completely supported the specification of the relationship between motivation and affect, largely supported the specification of the relationship between affect and engagement, but failed to support the specification of the relationship between motivation and engagement. The PISA data largely supported the specification of the mediation effects of affect on the relationship between motivation and engagement.

La présente analyse visait à tester le modèle dynamique (interactif) de l'affect, de la motivation et de l'engagement (Linnenbrink, 2007) dans l'enseignement des mathématiques avec un échantillon représentatif au niveau national. L'auto-efficacité, le concept de soi et l'anxiété liée aux mathématiques étaient des indicateurs de l'affect agréable et désagréable. La motivation intrinsèque et extrinsèque était des indicateurs de l'approche de la maîtrise et de la performance. La persistance éducative et l'activation cognitive étaient des indicateurs de l'engagement comportemental et cognitif. Le Programme international pour le suivi des acquis des élèves (PISA) de 2012 a fourni un échantillon de 4 978 élèves des États-Unis pour la modélisation des équations structurelles. Les résultats indiquent que les données PISA soutiennent globalement le modèle dynamique. Plus précisément, les données PISA ont complètement soutenu la spécification de la relation entre la motivation et l'affect, ont largement soutenu la spécification de la relation entre l'affect et l'engagement, mais n'ont pas soutenu la spécification de la relation entre la motivation et l'engagement. Les données PISA ont largement soutenu la spécification des effets de médiation de l'affect sur la relation entre la motivation et l'engagement.

Review of Literature

For this concise literature review, we adopted the *Achievement Goals Theory* as our overarching theoretical framework. This framework is a social-cognitive approach that emphasizes students' perceptions of and the interactions between cognition, affect, and behavior (Frith & Frith, 2012; Korman et al., 2015; Maehr & Zusho, 2009). In their attempts to achieve goals, all students strive to demonstrate competence in achievement contexts; thus, there is the element of cognition in the process to achieve goals. Success or failure depends on what students believe about their abilities and how they react to challenges in achieving goals; thus, there are also the elements of behavior and affect in the process to achieve goals. Cognitively, the purpose or reason why students pursue achievement goals is critical, speaking to motivation (Murayama & Elliot, 2019). Behaviorally, how students proceed to achieve goals, particularly when encountering difficulties in the process, matters critically, speaking to engagement (Luo et al., 2011). Finally, affective patterns, particularly after experiencing challenges, are critical in the process of achieving goals (Anderman & Wolters, 2006). Under this theoretical framework, there are specific manifestations of affect, motivation, and engagement in the process of doing well in mathematics. Indeed, there has been a strong recognition of the importance of the triangular effects of affect, motivation, and engagement in mathematics education (Linnenbrink, 2007).

Affect in Mathematics

Affect is a general term that encompasses three constructs: affective traits, emotions, and moods (e.g., Linnenbrink, 2006). According to the classic work of McLeod (1992), affect in mathematics is represented by beliefs, attitudes, and emotions. Beliefs involve the attribution of some sort of truth to systems of propositions or other cognitive configurations. Attitudes are orientations or predispositions toward a certain set of feelings. Emotions represent the rapidly-changing states of feelings experienced during certain activities in a particular context. Beliefs are the most stable and least intense, emotions are the most intense and least stable, and attitudes are intermediate on both dimensions.

Self-efficacy, self-concept, and anxiety (the opposite of self-confidence) are typical measures of beliefs about mathematics (e.g., Lebens et al., 2010; Linnenbrink & Pintrich, 2002; Meyer &Turner, 2006). Students' enjoyment of mathematics, feelings of helplessness in the learning of mathematics, and perceptions about the usefulness, relevance, and value of mathematics are typical measures of attitudes toward mathematics (e.g., Tapia & Marsh, 2000, 2005). Measuring emotions is not common, perhaps because when emotional responses become habitual or fixed, they function like attitudes (McLeod, 1992). Nonetheless, typical emotional responses to mathematics include joy and excitement when positive outcomes about mathematics occur and panic and frustration when negative outcomes about mathematics occur (e.g., Pekrun et al., 2007).

The role of affect in the learning of mathematics has received considerable attention from mathematics educators (Goldin et al., 2011; Leder & Grootenboer, 2005; Tapia & Marsh, 2000). Affect in mathematics is quite strongly related to the ability of students to learn new topics in mathematics, behave well in mathematics classes, and score high on mathematics tests (e.g., Boruchovitch, 2004). One of the major issues that mathematics educators face is the large proportion of students and adults who have negative beliefs, attitudes, and feelings about the subject (Nardi & Steward, 2002). The National Council of Teachers of Mathematics (2000) emphasized that students' confidence in and disposition toward mathematics are critical components in mathematics education.

Motivation in Mathematics

Motivation is the psychological feature that arouses a person to act in a way that moves that person toward a desired goal (Sansone & Harackiewicz, 2000). Motivation influences what, when, and how people learn (Vansteenkiste et al., 2006). In general, motivation consists of intrinsic and extrinsic motivation (Deci & Ryan, 2002) or a mastery goal orientation and a performance goal orientation (Eccles & Wigfield, 2002). From the mastery goal perspective, the purpose of learning is to grow in competence, master a task, improve in some way, and enjoy a challenge. From the performance goal perspective, the purpose of learning is to show individual ability, look competent, get recognition, and perform better than others. A mastery goal is intrinsic and task-oriented, whereas a performance goal is extrinsic and ability-oriented (Elliott & Story, 2017; Shatz, 2015).

The most direct way to measure motivation is to assess individual behaviors. In a classic manner, intrinsic motivation is measured through self-reports of interest in and enjoyment of learning activities, and extrinsic motivation is measured through self-reports of external reasons for putting effort into learning activities (e.g., Conti et al., 1995). Researchers generally measure the mastery goal by assessing whether a student's learning goals are established based on interests and curiosities as well as desires to learn content, gain knowledge, master materials, and overcome challenges (e.g., Elliot & Murayama, 2008). Researchers generally measure the performance goal by assessing the energization and direction of competence-related behaviors according to some external and internal standards of excellence (e.g., Elliot, 1997; Urdan, 1997).

Motivation has traditionally been a major concern among mathematics educators (Keys et al., 2012; Niepel et al., 2014). Motivated students show interest in learning activities, attend carefully to instruction, take notes to facilitate learning, work diligently to acquire new materials, feel confident about learning, and demonstrate persistence in difficult tasks, thus performing well in school as a result; whereas unmotivated students are likely to be inattentive during lessons and feel satisfactory when achieving minimum learning standards, thus falling behind in their studies as a result (Aunola et al., 2006; Schunk et al., 2008). Overall, there is a strong emphasis on the relationship between motivation and achievement in the learning of mathematics (e.g., Deci & Ryan, 2002).

Engagement in Mathematics

Engagement is an active behavior in the learning process, often defined as the amount of time and effort that students put into their studies and activities (Gonyea & Kuh, 2009). The traditional approach considers engagement as having two components: behavioral and emotional (e.g., Skinner & Belmont, 1993). Behavioral engagement is referred to as effort and persistence, and emotional engagement as positive and negative reactions to learning activities such as interest, boredom, happiness, sadness, and anxiety (Skinner et al., 2009). The contemporary approach conceptualizes engagement as comprising three components, with the addition of cognitive engagement that stresses an investment in learning activities that involves self-regulation or being strategic in learning (Fredricks et al., 2004; Jimerson et al., 2003).

Behavioral engagement has been measured by students' participation in, persistence in, avoidance of, and ignoring of their schoolwork (Gonida et al., 2009). Emotional engagement has been measured by a student's identification, sense of belonging to school, and positive attitude about learning (Marks, 2000). Cognitive engagement has been measured by a student's learning styles and learning strategies such as metacognitive and volitional strategies that promote self-regulated learning (i.e., students plan learning activities, exercise control over learning activities, and practice autonomy in the learning process) (Fredricks et al., 2004).

Student engagement is the most persistently identified factor for the improvement of students' overall learning experiences (Kuh et al., 2005; Ladd & Dinella, 2009; Martin & Rimm-Kaufman, 2015; Sciarra & Seirup, 2008). Specifically, in terms of the link between student engagement and academic performance, a recent meta-analysis concluded that all three dimensions of student engagement correlate positively with academic performance in primary and secondary school samples (Lei et al., 2018). The caution is that, in terms of their link with academic performance, there is a much greater certainty for the positive role of behavior engagement than cognitive engagement, whereas evidence may not be adequate for emotional engagement (Estévez et al., 2021).

The Dynamic Model of Affect, Motivation, and Engagement

Because of the importance of affect, motivation, and engagement, many studies have investigated the relationship among the three factors. For example, there is some evidence that affect enhances motivation (e.g., Hall et al., 2016) and shapes engagement (e.g., Linnenbrink-Garcia & Pekrun, 2011). These studies take on a one-on-one approach in that they focus on two from the three factors and examine the relationship between the two. As a matter of fact, little is known about whether and how affect, motivation, and engagement interact during the process in which students learn mathematics. Given how closely related these factors are to one another both conceptually and practically, the paucity of studies into their interaction in the learning of mathematics is surprising.

To fill this gap in the literature, Linnenbrink (2007) developed the dynamic model of affect, motivation, and engagement as shown in Figure 1. In this figure, Linnenbrink (2007) used "+" to indicate a position correlation or relationship and "−" to indicate a negative correlation or relationship. Linnenbrink (2007) also attempted to show the level of empirical support for each specification made in the model. Solid lines indicate consistent findings, and dashed lines indicate general patterns based on inconsistent findings. Arrows are employed to indicate direction or causality but the strength of each path is indicated by association or correlation.

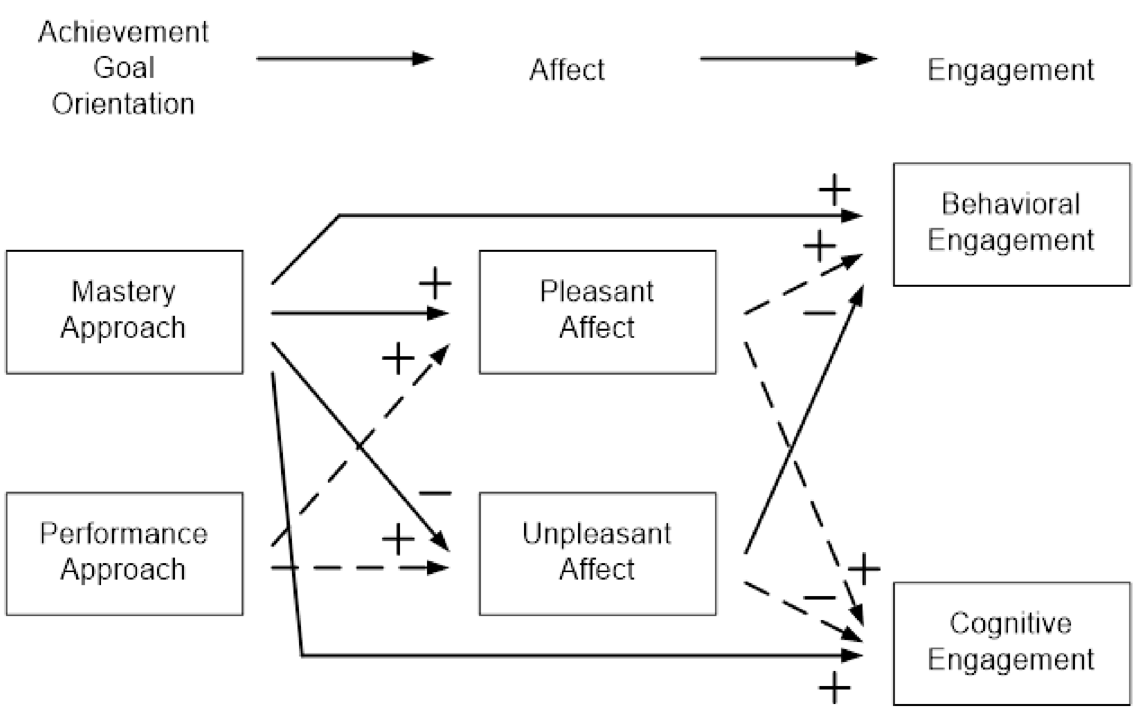
Linnenbrink's (2007) dynamic model of affect, motivation, and engagement is essentially a multi-dimensional mediation model in which motivation is the predictor, affect is the mediator, and engagement is the outcome. Specifically, motivation initiates the affective process of learning. Students with a mastery goal orientation actively learn and seek self-improvement, and students with a performance goal orientation attempt to demonstrate superior capability and performance. Affect interacts with motivation. The mastery goal orientation is positively associated with pleasant affect and negatively associated with unpleasant affect, and the performance goal orientation is either unassociated with or positively associated with both pleasant affect and unpleasant affect. Linnenbrink (2007) adopted behavior and cognition as engagement. There is a positive correlation between pleasant affect and increased behavioral engagement, and there is a negative correlation between unpleasant affect and decreased behavioral engagement. Also, pleasant affect correlates with more cognitive engagement, and unpleasant affect correlates with less cognitive engagement. Linnenbrink (2007) concluded the model by linking motivation with engagement. The mastery goal orientation motivates students to engage positively in terms of both behavior and cognition, and the performance goal orientation fails to make students see the need for either behavior engagement or cognitive engagement. Overall, Linnenbrink (2007) proposed that pleasant affect has a positive mediating function and unpleasant affect has a negative mediating function for the predictive effects of mastery goal orientation and performance goal orientation on behavioral engagement and cognitive engagement.

Many researchers view Linnenbrink's (2007) model as a good representation of the learning process, and as such they see an opportunity to link (part of) the model with academic performance. Researchers often propose their own theoretical models similar to Linnenbrink's and then link their models with academic performance. For example, Mega et al. (2014) tested their theoretical model that links emotions, self-regulated learning, and motivation to academic achievement (among undergraduate students). Emotions turn out to influence self-regulated learning and motivation which in turn influence academic performance, so as to demonstrate that self-regulated learning and motivation mediate the effects of emotions on academic performance. Similarly, according to Pekrun et al. (2009), achievement goals predict achievement emotions (e.g., anxiety, hopelessness) which in turn predict performance attainment, with seven out of eight emotions mediating the relationship between achievement goals and performance attainment.

No study has ever tested Linnenbrink's (2007) dynamic model of affect, motivation, and engagement directly in its entirety (without linking with academic performance); neither, even

Figure 1

The Dynamic (Interactive) Model of Affect, Motivation, and Engagement (Linnenbrink, 2007).



Note. Solid lines indicate consistent findings, and dashed lines indicate general patterns based on less consistent finding. The positive sign "+" indicates positive correlations, and the negative sign "–" indicates negative correlations. Copyright 2007 by Elsevier. Reprinted with permission.

more importantly, has any study utilized a nationally representative sample to test the model. As a result, such a significant theoretical advancement remains largely a research hypothesis. In the present analysis, we used nationally representative data from the Programme for International Student Assessment (PISA) to test the model. By assessing the extent to which the PISA data support the model, we were in a good position to refine or modify, if necessary, the model.

Conditions of Mediation

In general, four conditions need to be present for mediation. First, the predictor must be significantly related to the mediator. Second, the mediator must be significantly related to the outcome. Third, the predictor must be significantly related to the outcome. Finally, the relationship between the predictor and the outcome must be significantly reduced in the presence of the mediator (Baron & Kenny, 1986). Apart from testing the overall fit of the PISA data to the dynamic model of affect, motivation, and engagement, one critical aspect of our effort was to examine the fulfillment of the four conditions as a way to assess the mediation of affect on the relationship between motivation and engagement.

Method

Data

PISA is an international, large-scale standardized assessment that measures 15-year-old students in the domains of reading, mathematics, and science in a large number of countries and education systems. Rather than being limited to measuring the curriculum content that students have learned, the purpose of PISA is to measure the yield of different education systems to determine how well students who are approaching the end of mandatory education are prepared to meet challenges in the real world. PISA is administrated every three years to assess the students' level of knowledge and skills essential for full participation in society. Each cycle focuses on a major domain (e.g., mathematics) with detailed measures on many aspects of learning in the major domain, whereas the other two domains are addressed in less detail.

PISA 2012 is the latest cycle with a focus on mathematics, with the participation of 65 education systems (Organisation for Economic Co-operation and Development [OECD], 2013). PISA's stratified random sampling procedure included two stages. The first stage randomly sampled individual schools from each system, with probabilities proportional to their (enrollment) sizes. The second stage randomly sampled 35 eligible students in each selected school. In PISA 2012, students took paper and pencil tests on reading, mathematics, and science and answered a questionnaire about their homes, schools, and learning experiences. School principals also answered a questionnaire to provide information about their schools.

For the present analysis, we retrieved the national sample of 4978 students from the United States (US).[2] In this nationally-representative sample, 49% of the students were girls (2453) and 51% of the students were boys (2525), 79% of the students had at least one parent born in the US (3828) and 21% of the students had both parents born outside of the US (1002), 22% of the students had only one parent or guardian (982) and 78% of the students had two parents or guardians (3484), and 86% of the students spoke English at home (4196) and 14% of the students spoke another language at home (670). We utilized the questionnaire data of these students for the present analysis.

Variables

To test Linnenbrink's (2007) dynamic model where affect, motivation, and engagement interact in the learning process of mathematics, variables were sought to represent these latent constructs. There were two types of variables in PISA 2012, raw items or original responses and composite variables constructed by PISA, each based on a scale of items. PISA 2012 provided good indicators for measuring affect in mathematics from the perspective of belief about mathematics and attitude toward mathematics, very similar to McLeod's (1992) affective domain in mathematics. Specifically, belief about mathematics was measured by mathematics self-efficacy, which was students' convictions that they can successfully perform mathematical tasks, and mathematics self-concept, which was students' perceptions about their competence in mathematics (OECD, 2013). Attitude toward mathematics was measured by mathematics anxiety, which was students' feelings of helplessness and stress when working with mathematics (OECD, 2013). Appendix A describes these PISA data on affect in detail. Both mathematics self-efficacy and mathematics self-concept were composite variables, and we used them to represent pleasant affect. We used five raw items that were indicative of mathematics anxiety to represent unpleasant affect.[3] For both raw items and composite variables, a higher value indicated a lower pleasant affect and a higher unpleasant affect.

Although PISA 2012 did not provide any measures on motivation through mastery and performance goals, it did measure intrinsic and extrinsic motivations. Based on the connection between these different theoretical approaches (e.g., Elliott & Story, 2017; Shatz, 2015), PISA 2012 could still be utilized to approximate mastery and performance goals. We used four raw items that were indicative of intrinsic motivation to represent a mastery-goal orientation, and we used four raw items that were indicative of extrinsic motivation to represent a performance-goal orientation, as shown in Appendix A.[4] For all raw items, a higher value indicated a lower motivation.

In PISA 2012, there were five raw items indicative of behavioral engagement, based on the persistence of students on schooling tasks, similar to the theoretical approach that focuses on persisting when facing difficulties in academic work (e.g., Skinner et al., 2008). We used these five raw items to represent behavioral engagement. There were nine raw items indicative of cognitive engagement by means of cognitive activation, which was students' employment of cognitive strategies such as summarizing, questioning, and predicting when solving mathematics problems. Cognitive activation is very similar to cognitive engagement, often measured through metacognitive and volitional strategies (e.g., Yazzie-Mintz, 2007). We used these nine raw items to represent cognitive engagement. Appendix A describes these PISA data on engagement in detail. For all these raw items, a higher value indicated a lower engagement.

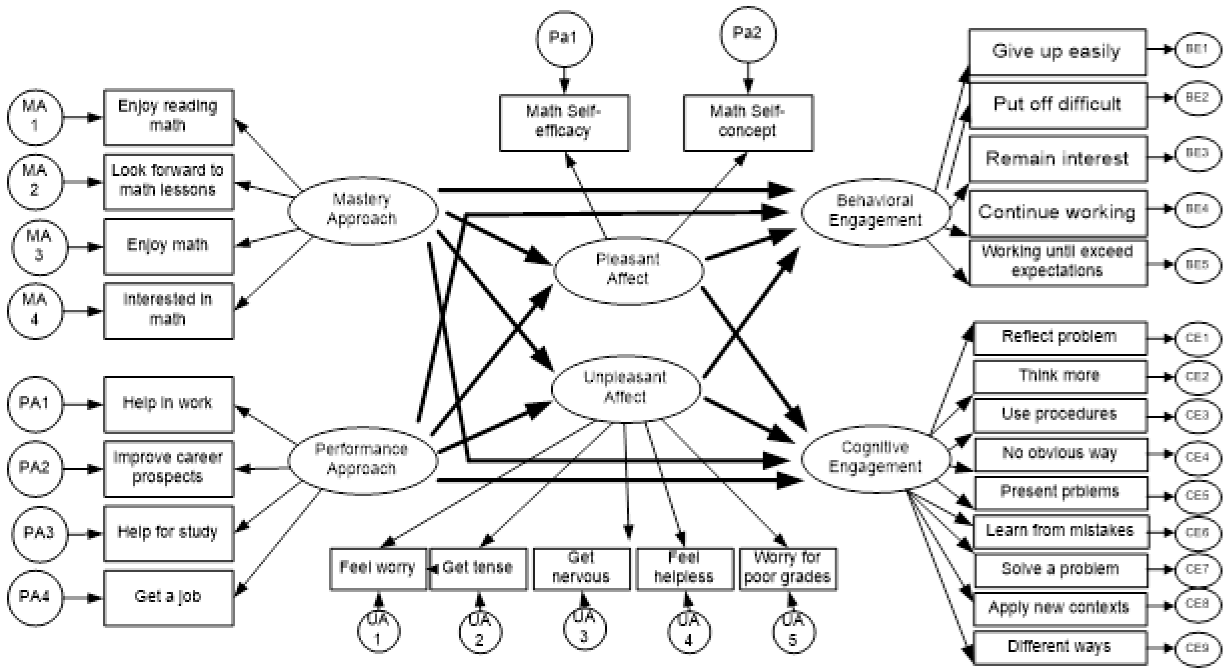
Overall, although PISA was not specifically designed to test the dynamic model of affect, motivation, and engagement (Linnenbrink, 2007), we believed that PISA 2012 provided sufficient information of affect, motivation, and engagement similar in nature to measures needed to test the dynamic model. As a result, we argued that PISA 2012 could be employed as a tool for testing the model.[5]

Model

Figure 1 that shows Linnenbrink's (2007) dynamic model of affect, motivation, and engagement in the learning of mathematics was operationalized into Figure 2. Figure 1 is a conceptual model

Figure 2

The Structural Equation Model Specifying the Relationships Among Students' Affect, Motivation, and Engagement



Note. This model operationalizes Linnenbrink (2007)

that integrates affect, motivation, and engagement in mathematics, whereas Figure 2 depicts a structural equation modeling (SEM) approach that operationalized Linnenbrink's (2007) theoretical framework. An SEM model contains a measurement model and a structural model.

In our measurement model, mathematics self-efficacy and mathematics self-concept were indicators for the latent variable of pleasant affect. The five items representing mathematics anxiety were indicators for the latent variable of unpleasant affect. The four items representing intrinsic motivation to learn mathematics were indicators for the latent variable of mastery approach in mathematics, and the four items representing extrinsic motivation to learn mathematics were indicators for the latent variable of performance approach in mathematics. There were five items as indicators for the latent variable of behavioral engagement, and there were nine items as indicators for the latent variable of cognitive engagement. This measurement model included a measurement error for each observed indicator. Our structural model included both directional relationships in the form of regression and nondirectional relationships in the form of correlation among latent variables of affect, motivation, and engagement in mathematics as shown in Figure 2. Mastery approach influences both pleasant affect and unpleasant affect, and so does performance approach. Additionally, mastery approach influences both behavioral engagement and cognitive engagement, and so does performance approach. Finally, pleasant affect influences both behavioral engagement and cognitive engagement, and so does unpleasant affect. This structural model included correlated measurement errors in circles.

Analysis

We employed SEM to test Linnenbrink's (2007) dynamic model of affect, motivation, and engagement in the learning of mathematics by examining the extent to which the model fits the data (Kaplan, 2009). If the fit is good, the directions and strengths of the paths would elucidate the structure of the theoretical model. If the fit is bad, alternative ways of specifying the model would modify the structure of the theoretical model. Data analysis consisted of a three-stage process (tests of statistical assumptions, confirmatory factor analysis [CFA], and SEM). The first two stages made sure the data were appropriate for SEM, and the last stage determined how well the model fit the data. We used SPSS to test statistical assumptions of multivariate normality, linearity, and multicollinearity considered critical to SEM (Vogt, 2007). Prior to SEM, CFA was performed separately on each construct in terms of affect, motivation, and engagement to ascertain acceptable measurement of the latent variables. Then SEM was used to test the validity of the hypothesized theoretical model.

Multiple indices are desirable to obtain a good triangulation of the model-data-fit for CFA and SEM (e.g., Byrne, 2012). Given that the traditional χ^2 statistic is affected by sample size, the overall fit of the model was also evaluated using the comparative fit index (CFI), Tucker-Lewis index (TLI), root mean square error of approximation (RMSEA), and standardized root mean square residual (SRMR), which are robust to sample size (e.g., Hu & Bentler, 1999). These indices are shown to be sufficient for measuring the fit of any SEM model with the data (Byrne, 2012; Hu & Bentler, 1999). The values of CFI and TLI lie between 0 and 1, with a value greater than .90 indicating acceptable fit (e.g., Browne & Cudeck, 1993). RMSEA values less than .05 are considered a good fit, values in the range of .06 and .08 are considered a moderate fit, and values greater than .10 indicate a poor fit (Browne & Cudeck, 1993). For SRMR, a value of less than .08 is considered a good fit (Browne & Cudeck, 1993). Model modification may occur when the original model does not fit the data. It involves adding or removing a path as suggested by the residuals and the modification indices obtained by running the original model (Hoyle, 1995). In the present analysis, both CFA and SEM were performed using Mplus.

Results

Descriptive statistics for all items used to measure affect, motivation, and engagement in mathematics are presented in Appendix A, together with skewness and kurtosis statistics that were used to assess univariate normality. All skewness statistics fell into the acceptable range, but kurtosis statistics indicated a somewhat deviation from normality for some items (Kline, 2016). As a result, a robust maximum likelihood estimator that is commonly applied when the assumption of normality is violated was utilized in both CFA and SEM. In addition, there were missing data in the present analysis, with a range from 2% to 36%. Missing data were treated with multiple imputation by using gender, family structure, immigrant status, and home language as auxiliary variables (Schlomer et al., 2010).

CFA was used to test the underlying structure of the latent variables of pleasant affect, unpleasant affect, mastery goal orientation, performance goal orientation, behavioral engagement, and cognitive engagement. The internal consistency or reliability was assessed using MacDonald's omega (ω) instead of Cronbach's alpha (α) (Geldhof et al., 2014). Table 1 presents the model-data-fit results for all seven scales. All model-data-fit statistics for each latent variable fell into the best fitting category; that is, statistically significant χ^2, CFI > .90, TLI > .90, SRMR <

Table 1

Model-Data-Fit Results of Confirmatory Factor Analysis (CFA) Models

Model	χ^2	*df*	CFI	TLI	SRMR	RMSEA [90% CI]	ω
Mathematics self-efficacy	686.446	34	.941	.905	.049	.050 [.045, .054]	.99
Mathematics self-concept	158.288	13	.983	.953	.034	.047 [.041, .054]	.97
Mathematics anxiety	86.745	13	.980	.944	.025	.048 [.039, .058]	.99
Mastery goal orientation	14.116	2	.998	.979	.004	.035 [.019, .053]	.99
Performance goal orientation	156.800	14	.980	.961	.037	.045 [.039, .052]	.91
Behavioral engagement	25.882	2	.994	.914	.010	.049 [.033, .067]	.97
Cognitive engagement	274.717	22	.968	.920	.026	.048 [.043, .053]	.86

Note. All χ^2 statistics are significant at the level of .001. CFI = comparative fit index. TLI = Tucker-Lewis index. SRMR = standardized root mean square residual. RMSEA = root mean square error of approximation. CI = confidence interval.

Table 2

Model-Data-Fit Results of Structural Equation Modeling (SEM) Models

Model	χ^2	*df*	CFI	TLI	SRMR	RMSEA [90% CI]
M0	48502.464	780	.000	.000	.273	.111 [.110, .112]
M1	3879.381	590	.931	.909	.061	.033 [.032, .034]
M2	2781.872	585	.954	.939	.039	.027 [.026, .028]

Note. All χ^2 statistics are significant at the level of .001. CFI = comparative fit index. TLI = Tucker-Lewis index. SRMR = standardized root mean square residual. RMSEA = root mean square error of approximation. M0 = the null model. M1 = the full model. M2 = the mega model. CI = confidence interval.

.08, and RMSEA < .05. In addition, all latent constructs demonstrated excellent internal reliabilities in Table 1. Overall, the results indicated that the measurement model for each latent variable was adequate, laying the foundation for the SEM analysis to test the dynamic model of affect, motivation, and engagement in mathematics.

A series of SEM models were developed to examine the extent to which PISA 2012 data support Linnenbrink's (2007) dynamic or interactive model of affect, motivation, and engagement in mathematics. A null model, M0, was used as the baseline model in which all structural paths were zero and all measurement paths from latent variables to observed indicators were 1. Next, a full model, M1, was established by adding paths connecting the latent variables, as shown in Figure 2, to assess how well the predicted interrelationships between affect, motivation, and engagement matched the hypothesized structural model. The full model tested the direct effects between affect, motivation, and engagement as well as the mediation of affect on the relationship between motivation and engagement. The full model in Table 2 suggested a very reasonable fit to the PISA data. Specifically, χ^2 = 3879.381, *df* = 590, *p* < .001; CFI = .931; TLI = .909; SRMR = .061; and RMSEA = .033 with a 95% CI of [.032, .034]. Furthermore, all the standardized residual correlations were less than |1|, indicating good local fit (Kline, 2016). Finally, a mega model, M2, was established to compare with the full model, M1, by separating pleasant affect into two latent constructs based on self-efficacy and self-concept. The rationale was that all other latent variables such as unpleasant affect used individual raw items as indicators but pleasant affect used composite variables of self-efficacy and self-concept as indicators. The mega model suggested a very reasonable fit to the PISA data. Specifically, $\chi^2 = 2781.872$, $df = 585$, $p < .001$; CFI = .954; TLI = .939; SRMR = .039; RMSEA = .027 with a 95% CI of [.026, .028]. M1 and M2 were very similar in terms of fit statistics. M2 provided further evidence that M1 specified a more parsimonious structure. This result, in conjunction with the increase in fit for all indicators compared with M0, suggested that the full model, M1, explained the data and that Linnenbrink's (2007) dynamic model of affect, motivation, and engagement in mathematics was supported by the PISA 2012 data.[6] The interpretation of results would logically focus on M1. Table 3 presents

Table 3

Estimates of Path Coefficients and Mediation Effects

Parameter	Estimate	SE	95% CI	S. Estimate
Mastery approach ON pleasant affect	.39*	.05	[.34, .44]	.85
Performance approach ON pleasant affect	-.00	.06	[-.06, .05]	-.01
Mastery approach ON unpleasant affect	-.82*	.05	[-.95, -.69]	-.78
Performance approach ON unpleasant affect	.09	.07	[-.05, .23]	.09
Pleasant affect ON behavioral engagement	.79*	.09	[.52, 1.05]	.62
Unpleasant affect ON behavioral engagement	.08*	.06	[.02, .13]	.14
Pleasant affect ON cognitive engagement	.34*	.07	[.06, .61]	.16
Unpleasant affect ON cognitive engagement	.07	.04	[-.00, .14]	.08
Mastery approach ON behavioral engagement	-.07	.08	[-.16, .03]	-.12
Performance approach ON behavioral engagement	.10*	.04	[.05, .14]	.17
Mastery approach ON cognitive engagement	.04	.07	[-.09, .18]	.05
Performance approach ON cognitive engagement	.20*	.05	[.11, .29]	.21
Pleasant affect IND mastery approach and behavioral engagement	.30*	.09	[.19, .41]	.53
Pleasant affect IND performance approach and behavioral engagement	-.00	.04	[-.04, .04]	-.01
Pleasant affect IND mastery approach and cognitive engagement	.13*	.06	[.02, .24]	.14
Pleasant affect IND performance approach and cognitive engagement	-.00	.01	[-.02, .02]	-.00
Unpleasant affect IND mastery approach and behavioral engagement	-.06*	.04	[-.11, -.01]	-.11
Unpleasant affect IND performance approach and behavioral engagement	.01	.01	[-.01, .02]	.01
Unpleasant affect IND mastery approach and cognitive engagement	-.06	.03	[-.12, .00]	-.06
Unpleasant affect IND performance approach and cognitive engagement	.01	.01	[-.01, .02]	.01

Note. Estimate = unstandardized path coefficients. SE = standard error. CI = confidence interval. S. Estimate = standardized path coefficients. ON = direct effects (e.g., the direct effects of mastery approach on pleasant affect). IND = indirect effects (e.g., the indirect effects of pleasant affect on the relationship between mastery approach and behavioral engagement).

various statistical estimates for all of the paths in M1. It is customary to interpret standardized estimates when variables are continuous (Kelloway, 2015).

The direct effects can be interpreted as regression coefficients in standardized forms. A standardized coefficient is considered strong when larger than .70, moderate when larger than .50, and weak when smaller than .30 (Brown, 2006; Saris et al., 2009). Statistically significant coefficients ranged from .11 to .85 in absolute value. The direct effects of mastery approach on both pleasant affect (effects = .85, SE = .05, $p < .001$) and unpleasant affect (effects = -.78, SE = .05, $p < .001$) were strong. A one standard deviation or SD increase in mastery approach was related to an increase of .85 SD in pleasant affect and a decrease of .78 SD in unpleasant affect. The direct effects of pleasant affect on behavioral engagement were moderate (effects = .62, SE = .09, $p < .001$), whereas the direct effects of pleasant affect on cognitive engagement (effects = .16, SE = .07, $p = .02$), the direct effects of unpleasant affect on behavioral engagement (effects = .14, SE = .06, $p = .01$), and the direct effects of performance approach on both behavioral engagement (effects = .17, SE = .04, $p < .001$) and cognitive engagement (effects = .21, SE = .05, $p < .001$) were weak. These effects can be interpreted in a similar manner.

With the same metric, the indirect effects of pleasant affect on the relationship between mastery approach and behavioral engagement were moderate (effects = .53, SE = .09, $p < .001$). A one SD increase in pleasant affect increased the effects of mastery approach on behavioral engagement by .53 SD. The indirect effects of pleasant affect on the relationship between mastery approach and cognitive engagement (effects = .14, SE = .06, $p < .02$) and the indirect effects of unpleasant affect on the relationship between mastery approach and behavioral engagement (effects = -.11, SE = .04, $p < .01$) were weak. These effects can be interpreted in a similar manner. Overall, both pleasant affect and unpleasant affect had a complete mediation on the relationship between mastery approach and behavioral engagement (i.e., the effects of X on Y decreases to zero with the inclusion of mediator). In addition, pleasant affect had a complete mediation on the relationship between mastery approach and cognitive engagement.

Discussion

Summary of Principal Findings

The goal of the present analysis was to test, with nationally representative data, the dynamic model of affect, motivation, and engagement in the domain of mathematics education as theorized in Linnenbrink (2007). The overall conclusion is that the nationally representative data supported the dynamic model. To facilitate the summary of principal findings, a path-to-path comparison was presented in Table 4 to illustrate the degree to which the model fit the data. Specifically, the PISA data confirmed 4 out of 4 specifications concerning the relationship between motivation and affect in Linnenbrink (2007). This is a complete support of the dynamic model with respect to the relationship between motivation and affect. The PISA data confirmed 3 out of 4 specifications concerning the relationship between affect and engagement in Linnenbrink (2007). This is a nearly complete support of the dynamic model with respect to the relationship between affect and engagement. The PISA data confirmed 0 out of 4 specifications about the relationship between motivation and engagement in Linnenbrink (2007). The present analysis does not support the way that motivation is related to engagement in Linnenbrink (2007).

The PISA data confirmed 2 out of 4 specifications concerning the mediation effects of pleasant affect on the relationship between motivation and engagement in Linnenbrink (2007). This is a

Table 4

Summary of Findings in Comparison with the Dynamic Model of Affect, Motivation, and Engagement (Linnenbrink, 2007)

Parameters	Current Results	Linnenbrink (2007)
Mastery approach ON pleasant affect	Yes (+)	Yes (+)
Performance approach ON pleasant affect	No (0)	No (0)
Mastery approach ON unpleasant affect	Yes (−)	Yes (−)
Performance approach ON unpleasant affect	No (0)	No (0)
Pleasant affect ON behavioral engagement	Yes (+)	Yes (+)
Unpleasant affect ON behavioral engagement	Yes (+)	Yes (−)
Pleasant affect ON cognitive engagement	Yes (+)	Yes (+)
Unpleasant affect ON cognitive engagement	No (0)	No (0)
Mastery approach ON behavioral engagement	No (0)	Yes (+)
Performance approach ON behavioral engagement	Yes (+)	No (0)
Mastery approach ON cognitive engagement	No (0)	Yes (+)
Performance approach ON cognitive engagement	Yes (+)	No (0)
Pleasant affect IND mastery approach and behavioral engagement	Yes (+)	No (0)
Pleasant affect IND performance approach and behavioral engagement	No (0)	No (0)
Pleasant affect IND mastery approach and cognitive engagement	Yes (+)	No (0)
Pleasant affect IND performance approach and cognitive engagement	No (0)	No (0)
Unpleasant affect IND mastery approach and behavioral engagement	Yes (−)	Yes (−)
Unpleasant affect IND performance approach and behavioral engagement	No (0)	No (0)
Unpleasant affect IND mastery approach and cognitive engagement	No (0)	No (0)
Unpleasant affect IND performance approach and cognitive engagement	No (0)	No (0)

Note. Under Current Results, Yes = statistically significant, No = Not statistically significant. Under Linnenbrink (2007), Yes = Specified, and No = Not specified. In both columns, (+) = Positive relationship, (−) = Negative relationship, and (0) = No relationship.

partial support of the dynamic model with respect to pleasant affect mediating the relationship between motivation and engagement. The PISA data confirmed 4 out of 4 specifications concerning the mediation effects of unpleasant affect on the relationship between motivation and engagement in Linnenbrink (2007). This is a complete support of the dynamic model with respect to unpleasant affect mediating the relationship between motivation and engagement.

Theoretical Implications

The present analysis is likely the first empirical attempt to test the dynamic model of affect, motivation, and engagement in mathematics education (Linnenbrink, 2007). Without any previous evidence to compare with, we focus on some alternative explanations of our results, particularly for those that contradicted the model specifications. Linnenbrink (2007) specified that unpleasant affect was negatively correlated with behavioral engagement. This correlation was positive according to the PISA data. Students who experience unpleasant affect can still be persistent and effortful in their learning tasks and thus engaged. The alternative explanation may focus on the determination of students to achieve a certain goal. Linnenbrink's (2007) specification may still be true, without a consideration of the degree of determination. If students strongly set their mind to do something, they can endure some unpleasant feelings during the process. If determination comes close conceptually to motivation, then motivation may also moderate the relationship between affect and engagement. This finding in general adds to Boruchovitch (2004) that both pleasant affect and unpleasant affect can positively correlate with behavioral engagement. This finding may also suggest a way to overcome negative effects of unpleasant affect on behavioral engagement (Nardi & Steward, 2002) by tapping into students' determination to achieve a certain academic goal.

Linnenbrink (2007) specified that mastery approach was positively correlated with both behavioral engagement and cognitive engagement. These correlations were not statistically significant according to the PISA data. Students who learn mathematics because of their interests, their desires for growth in competence, and their enjoyment of challenges may not necessarily engage themselves either behaviorally or cognitively in their learning tasks. The alternative explanation here may connect with our earlier discussion. If motivation moderates the relationship between affect and engagement as discussed earlier, then motivation may not connect directly with engagement, or motivation may connect with engagement only weakly as suggested by the lack of statistical significance in the present analysis. This finding in general adds complicity to Aunola et al. (2006) and Schunk et al. (2008) that even motivated students may not demonstrate either behavioral engagement or cognitive engagement. There is a possibility that motivation may just be a moderator in function.

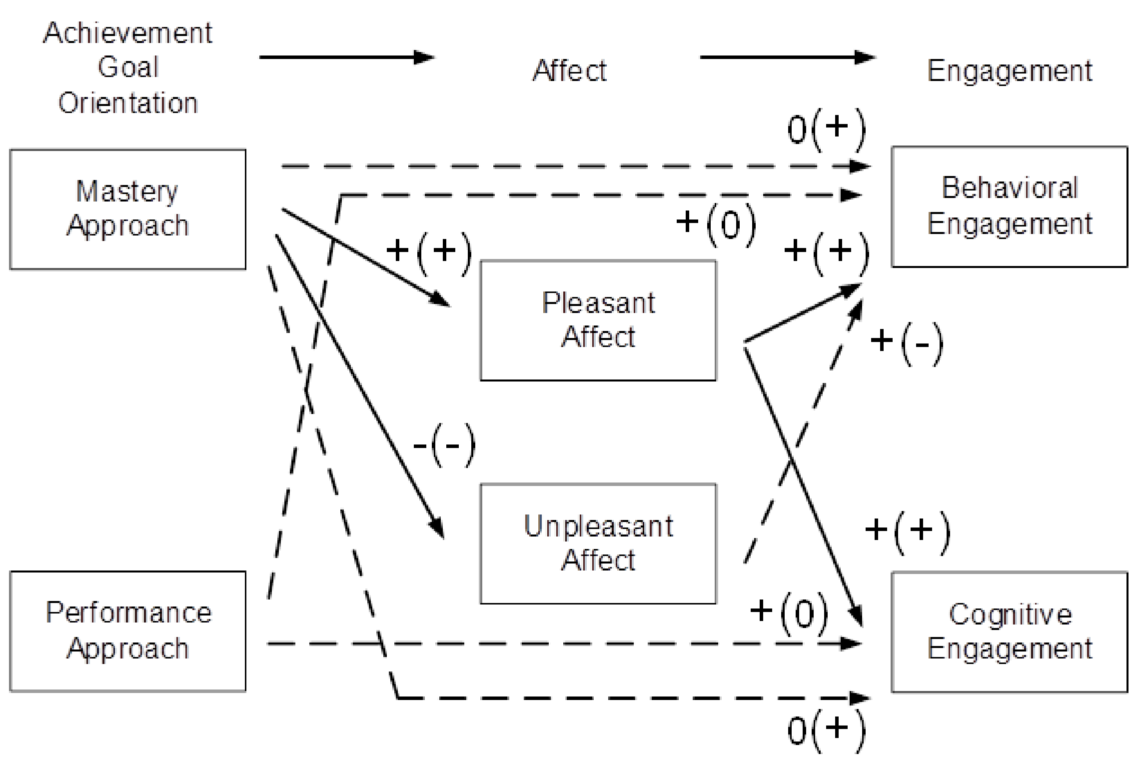
Linnenbrink (2007) specified that performance approach was not correlated with either behavioral engagement or cognitive engagement. These correlations were statistically significant according to the PISA data. Students who perceive mathematics to be useful to them and to their future studies and careers may still engage both behaviorally and cognitively in their learning tasks. The alternative explanation may also relate to the determination of students as we discussed earlier. Again, Linnenbrink's (2007) specification may still be true, without a consideration of the degree of determination. If students truly believe that mathematics is their "passport" to a prosperous career such as computer game programming, why would they not engage both behaviorally and cognitively to secure their passport? Alternatively, it may also be possible for performance approach to increase to correlate with both types of engagement. This finding in general reinforces Elliott and Story (2017) that, concerning academic wellbeing of students, extrinsic motivation cannot be underestimated. Students may be willing to "suffer" with extrinsic reasons as a means to an end even though they are not intrinsically convinced of any worthiness.

Linnenbrink (2007) specified that pleasant affect does not mediate the effects of mastery approach on either behavioral engagement or cognitive engagement. The PISA data revealed that pleasant affect positively mediated the effects of mastery approach on both behavioral engagement and cognitive engagement. Students who experience more pleasant affect may show stronger effects of motivation on both behavioral engagement and cognitive engagement. The alternative explanation is that pleasant affect may reinforce motivation; that is, positive feelings during the process may inform students that their motivated goals are actually achievable or hopeful. In other words, stronger effects of motivation on engagement can be obtained by increased pleasant affect functioning as positive reinforcement for motivation. This finding in general enriches McLeod (1992) and Goldin et al. (2011) on the importance of affect in mathematics education. We suggest that pleasant affect can also positively mediate the effects of motivation, especially mastery approach, on both types of engagement. Overall, these discussions so far may suggest that the relationship among motivation, affect, and engagement may be far more complex than what Linnenbrink (2007) theorized about them originally.

Because we utilized a nationally representative database, we are in a very good position to modify Linnenbrink's (2007) dynamic model. Overall, the model can be tentatively revised as shown in Figure 3. The positive signs indicate positive effects, and the negative signs indicate negative effects. The paths and specifications that differ from Linnenbrink (2007) are shown by the dotted lines, and the solid lines indicate agreement. This revised dynamic model is tentative in that it is offered as a new research hypothesis in expectation for further confirmatory studies.

Figure 3

Revised Dynamic Model Linking Affect, Motivation, and Engagement from Linnenbrink (2007).



Note. Dotted lines indicate disagreement with Linnenbrink's model, and solid lines indicate agreement with Linnenbrink's model. Positive signs with parentheses (+) indicate positive relationships from Linnenbrink's model, and negative signs with parentheses (–) indicate negative relationships from Linnenbrink's model. Positive signs without parentheses + indicate positive relationships from the present analysis, and negative signs without parentheses – indicate negative relationships from the present analysis. Finally, 0 indicates no relationship concerning a path.

Practical Implications

The present analysis completely confirmed Linnenbrink's (2007) specifications about the relationship between motivation and affect. The implication is that better motivation and better affect go hand in hand. In the teaching and learning of mathematics, educators may use mastery approach as a way to promote students' pleasant affect and to reduce their unpleasant affect. Given that pleasant affect was measured by self-efficacy and self-concept and that unpleasant affect was measured by mathematics anxiety, mathematics educators may use their daily interactions with students to purposefully boost students' motivation to learn mathematics based on interests, desires for growth in competence, and enjoyment of challenges to help students develop positive self-efficacy and self-concept and manage mathematics anxiety.

The present analysis was in a nearly complete position to support Linnenbrink's (2007) specifications about the relationship between affect and engagement. The implication is that better affect and better engagement go hand in hand. Specifically, mathematics educators may use pleasant affect to engage students both behaviorally and cognitively by striving to help students develop their self-efficacy and self-concept. Perhaps surprising to mathematics educators, unpleasant affect may also engage students behaviorally. Some appropriate level of mathematics anxiety can also enhance persistence and effort in the learning of mathematics.

The present analysis completely confirmed Linnenbrink's (2007) specifications about the mediation effects of unpleasant affect on the relationship between mastery approach and behavioral engagement. The implication is that reducing students' mathematics anxiety may increase the influence of motivation on persistence. Mathematics educators are encouraged to make the reduction of students' mathematics anxiety an instructional goal and integrate strategies for reducing mathematics anxiety into their classroom practice. There are some strategies in the literature that show effectiveness in reducing students' mathematics anxiety such as systematic desensitization training, self-management of emotional stress, and systematic skill building that makes students demonstrate what they can already do and what they need to do next.

Limitations

First, because PISA is not designed for testing Linnenbrink's (2007) dynamic model of affect, motivation, and engagement, the main limitation of the present analysis comes from the characteristics of the PISA data. The measures in PISA may not exactly match or capture the constructs in Linnenbrink (2007), concerning affect, motivation, and engagement. This issue is evident with respect to almost every major construct. Second, PISA works with the population of 15-year-old students. Although a PISA sample is nationally representative of the population, the data and the results are limited to this specific age group. Linnenbrink (2007) was not very specific about age concerning the dynamic model. In Linnenbrink (2007), empirical evidence comes from a wide range of student populations including upper elementary, middle school, high school, and college students. This fact may limit the generalizability of results from the present analysis. The last limitation relates to the nature of a cross sectional data analysis which cannot verify any causal processes and relationships among factors or constructs. Although SEM analyses can provide some suggestive support for putative causal models, longitudinal research will ultimately be needed to delineate more clearly the causal processes that link affect, motivation, and engagement.

Suggestions

First, given that the U.S. sample was largely positive for the dynamic model of affect, motivation, and engagement as specified in Linnenbrink (2007), we suggest that the research opportunity is mature to extend the testing to other cultures or countries as we discussed in Note 2. Cultural consistency or inconsistency may further reveal the nature of the dynamic model. Second, to align completely with the dynamic model, the present analysis did not include student characteristics in any SEM model. Nonetheless, individual differences in terms of gender, race, socioeconomic status, and family structure for example may exist in affect, motivation, and engagement. These variables have the potential to function as confounding factors that may impact the behaviors of the dynamic model. Future research may control for student characteristics in the dynamic model. Third, future studies may use comprehensive measures for affect, motivation, and engagement to fully operationalize the constructs in Linnenbrink (2007). This suggestion speaks to the need to conduct research studies that are specifically designed to test Linnenbrink's (2007) dynamic model. Fourth, future studies may confirm the dynamic model using data collected from different age groups to allow for further generalization of the dynamic model. Further analyses ideally may compare the dynamic model across multiple age groups to test the invariance of the dynamic model. The invariance speaks to the same structural paths among affect, motivation, and engagement across multiple age groups. Finally, given that a major component of the dynamic model concerns mediation, the new development in mediation analysis (e.g., Rucker et al., 2011) may offer a fresh opportunity to examine the dynamic model. Further insights may occur from the new approach to enhance our understanding of the dynamic model.

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Notes

1. This paper draws on data collected by Shanshan Hu in completion of a manuscript Doctor of Philosophy (PhD) at the University of Kentucky: Hu, S. (2018). *Affect, motivation, and engagement in the context of mathematics education: Testing a dynamic model of interactive relationships*. Theses and Dissertations—Educational, School, and Counseling Psychology. 71. https://doi.org/10.13023/etd.2018.234
2. All of the studies that led to the formulation of Linnenbrink's (2007) dynamic model of affect, motivation, and engagement were conducted in the context of the United States. In recognition of the social, cultural, and educational diversities even between neighboring countries such as Canada and the United States, we thought it was reasonable to examine the model first with the national sample of the United States. If the outcome is positive, our motivation increases to extend the testing to other countries.
3. When looking at indicators for positive or pleasant affect and negative or unpleasant affect, those for positive affect appeared to be general and those for negative affect appeared to be specific. The reason was that indicators for positive affect, mathematics self-efficacy and mathematics self-concept, were composite variables, whereas indicators for negative affect were raw items. The composite variables were constructed, each based on a scale of raw items. We did not use the raw items for mathematics self-efficacy and mathematics self-concept directly as indicators for positive affect. The reason was that if we had done so, it would have been necessary to build a confirmatory factor analysis on that part of the SEM so as to specify two different latent constructs (i.e., mathematics self-efficacy and mathematics self-concept). The resulting SEM would have been too complicated to be estimated.
4. With merits, some scholars may consider enjoyment as measuring emotion instead of mastery goal. As such, the two enjoyment items may be moved from mastery goal to pleasant affect. We avoided this practice for two reasons. One was of measurement. As we discussed in our literature review, mastery goal is of intrinsic motivation, and enjoyment is a key indicator of intrinsic motivation. The other was of modeling. The two indicators for pleasant affect, mathematics self-efficacy and mathematics self-concept, were composite variables, whereas the two indicators for enjoyment were raw items. It is not common in the research literature to couple raw items with composite variables together as indicators of one latent construct. The solution would be to bring in raw items for the two composite variables so that all indicators would be raw items. In doing so, however, we would run into tremendous difficulties in modeling as discussed in the third note above.
5. As we discussed in our literature review, there have not been any studies that test Linnenbrink's (2007) dynamic model of affect, motivation, and engagement directly in its entirety. In our literature review, we presented some studies that proposed their own theoretical models resembling some ideas in Linnenbrink's (2007) model and then often linked their models with academic performance that is not in Linnenbrink's (2007) model. In the present analysis, we exclusively examined Linnenbrink's (2007) model as is.
6. In SEM, standardized residual variance, which refers to the variance of the observed variables that is not explained by the latent constructs is commonly used to provide some insights into the total variance explained for the dependent variables. In the present analysis, standardized residual variance for M1 ranged from .19 to .90, with the majority less than .50. This fact also indicated that M1 was an adequate model.

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Appendix A: Description of Items Measuring Affect, Motivation, and Engagement in Mathematics with Descriptive Statistics

Variable	M	SD	Skewness	Kurtosis
Mathematics self-efficacy (for pleasant affect)				
Using a train timetable to work out how long it would take to get from one place to another.	3.07	.77	-.50	-.18
Calculating how much cheaper a TV would be after a 30% discount.	3.09	.84	-.55	-.48
Calculating how many square meters of tiles would be needed to cover a floor.	3.01	.84	-.44	-.56
Understanding graphs presented in newspapers.	3.22	.77	-.77	.15
Solving equations like 3x+5=17.	3.63	.64	-1.84	3.41
Finding the actual distance between two places on a map with a 1:10000 scale.	2.68	.92	-.06	-.91
Solving equations like 2(x+3) = (x+3)(x-3).	3.29	.84	-1.03	.32
Calculating the petrol-consumption rate of a car.	2.92	.85	-.33	-.65
(1 = not at all confident, 2 = not very confident, 3 = confident, 4 = very confident)				
Mathematics self-concept (for pleasant affect)				
I am just not good at mathematics.*	2.77	.95	-.44	-.68
I get good grades in mathematics.	2.98	.77	-.50	.00
I learn mathematics quickly.	2.72	.89	-.21	-.71
I have always believed that mathematics is one of my best subjects.	2.50	1.07	.02	-1.24
In my mathematics class, I understand even the most difficult work.	2.47	.91	.01	-.79
(1 = strongly disagree, 2 = disagree, 3 = agree, 4 = strongly agree)				
Mathematics anxiety (for unpleasant affect)				
I often worry that it will be difficult for me in mathematics classes.	2.64	.89	-.13	-.74
I get very tense when I have to do mathematics homework.	2.29	.90	.32	-.62
I get very nervous doing mathematics problems.	2.15	.83	.45	-.24
I feel helpless when doing a mathematics problem.	2.01	.84	.66	.02
I worry that I will get poor grades in mathematics.	2.48	1.01	.04	-1.09
(1 = strongly agree, 2 = agree, 3 = disagree, 4 = strongly disagree)				
Intrinsic motivation (for mastery goal orientation)				
I enjoy reading about mathematics.	2.19	.83	.26	-.53
I look forward to my mathematics lessons.	2.43	.89	.09	-.73
I do mathematics because I enjoy it.	2.27	.94	.32	-.76
I am interested in the things I learn in mathematics.	2.51	.89	.01	-.73
(1 = strongly disagree, 2 = disagree, 3 = agree, 4 = strongly agree)				

Variable	M	SD	Skewness	Kurtosis
Extrinsic motivation (for performance goal orientation)				
Making an effort in mathematics is worth it because it will help me in the work that I want to do later on.	3.07	.79	-.67	.20
Learning mathematics is worthwhile for me because it will improve my career prospects and chances.	3.04	.82	-.73	.26
Mathematics is an important subject for me because I need it for what I want to study later on.	2.90	.89	-.46	-.54
I will learn many things in mathematics that will help me get a job.	3.05	.81	-.71	.18
(1 = strongly disagree, 2 = disagree, 3 = agree, 4 = strongly agree)				
Behavioral engagement				
When confronted with a problem, I give up easily.*	3.86	1.00	-.84	.45
I put off difficult problems.*	3.43	1.10	-.35	-.50
I remain interested in the tasks that I start.	3.59	.99	-.48	-.06
I continue working on tasks until everything is perfect.	3.60	1.08	-.38	-.57
I continue working on tasks until it exceeds expectations.	3.37	1.09	-.17	-.62
(1 = not at all like me, 2 = not much like me, 3 = somewhat like me, 4 = mostly like me, 5 = very much like me)				
Cognitive engagement				
The teacher asks questions that make us reflect on the problem.	2.92	.88	-.36	-.72
The teacher gives problems that require us to think for an extended time.	2.94	.85	-.31	-.70
The teacher asks us to decide on our own procedures for solving complex problems.	2.46	.98	.08	-1.01
The teacher presents problems for which there is no immediately no obvious method of solution.	2.65	.93	-.08	-.88
The teacher presents problems in different contexts so that students know whether they have understood concepts.	2.91	.89	-.36	-.75
The teacher helps us to learn from mistakes we have made.	3.05	.93	-.61	-.63
The teacher asks us to explain how we have solved a problem.	3.16	.88	-.71	-.45
The teacher presents problems that require students to apply what they have learned to new contexts.	3.10	.86	-.56	-.57
The teacher gives problems that can be solved in several different ways.	2.94	.86	-.35	-.71
(1 = never or rarely, 2 = sometimes, 3 = often, 4 = always or almost always)				

Note. * indicates scale reverse.