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Solo Based-Cognition Levels of Inductive Reasoning in Geometry

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Inductive reasoning is one of the essential forms of thinking in mathematical discoveries and is useful in the workplace of any field. This investigation examines and characterizes the cognition levels and developmental trends in the inductive reasoning of 27 top-performing prospective mathematics teachers from three universities in Cebu City, Philippines. Participants were given a test questionnaire to assess the use of inductive task items. An in-depth analysis of the solution processes and the utilization of clustering through percentile quartiles generated two levels of cognition that support the SOLO taxonomy model. The levels are derived from a modified checklist that was based on the SOLO taxonomy. The findings support a trend of cognition for inductive reasoning ability in geometry and suggest that incoherence and the inability to connect learning to new task obstructs success. Participants who were unsuccessful in lower level tasks were not effective in higher level tasks.

Le raisonnement inductif est une des formes essentielles de pensée dans les découvertes en mathématiques, et est utile dans les milieux de travail de n'importe quel domaine. Cette étude analyse et caractérise les niveaux de cognition et les tendances en raisonnement inductif chez 27 des meilleurs futurs enseignants de mathématiques de trois universités à Cebu City, Philippines. Les participants ont complété un questionnaire visant l'évaluation de l'emploi des items impliquant un raisonnement inductif. Une analyse approfondie des processus de solution et de l'emploi du regroupement par des percentiles et des quartiles a généré deux niveaux de cognition qui appuient le modèle de la taxonomie SOLO sur les objectifs d'apprentissage. Les niveaux découlent d'une liste de vérification modifiée et basée sur la taxonomie SOLO. Les résultats appuient une tendance en cognition portant sur la capacité de raisonner de façon inductive en géométrie, et portent à croire que l'incapacité à lier l'apprentissage aux nouvelles tâches entrave la réussite. Les participants qui ne réussissaient pas les tâches de niveau bas n'étaient pas performants avec les tâches de niveau plus élevé.

Cognition or thinking comes from the Latin word *cognoscere* which means "to get to know." It is a mental process present in reasoning. It includes mental processes like perceiving, remembering, understanding, as well as the ability to apply previously aquired knowledge and available information to justify claims (Ashcraft, 1994). Inductive and deductive reasoning are principal processes that are also essential for the expansion and the application of mathematics to life problems (Barody, 1993; Bennett & Nelson, 1998; Kelly, 1956; Sheffield & Cruikshank, 1996; Sonnabend, 1997; Wood, Wood, & Boyd, 2005). In performing mathematical inductive reasoning tasks, conceptual understanding and background knowledge is vital (Kemp & Jern, 2013).

Human cognition is based on a unique talent of generalizing knowledge from a few specific examples (Griffiths, Chater, Kemp, Perfors, & Tenenbaum, 2010) thereby inferring or performing an induction. This activity of the mind is sometimes called reasoning by induction, or inductive inference (Bennett & Nelson, 1998; Sheffield & Cruikshank, 1996; Sonnabend, 1997), or conjecture (Barody, 1993). This thinking activity is relevant to almost all areas of cognition (Kemp & Jern, 2013) and is considered a fundamental component of thinking (Csapó, 1997). Inductive reasoning has been studied by researchers from many fields (Kemp & Jern, 2013) and is one of the most broadly studied processes of cognition with its own theoretical foundations (Csapó, 1997). For instance, a prescriptive theory of inductive reasoning proposed by Klauer and Phye (2008) identified cognitive processes using a procedural strategy for making comparisons and for which conceptual coherence was considered necessary in inductive inference, and perhaps independent of a particular concept (Coley, Hayes, Lawson, & Moloney, 2004). Coley et al. (2004) hypothesized that there is no basic level of cognition and proposed that different hierarchical levels are required for a variety of conceptual functions in adults. Griffiths et al. (2010) introduced a theory based on Bayesian models for induction that predicted optimal inference and explained why human generalization works at the computational level, and how it can be achieved given only scant data.

These studies of human cognition have revealed the importance of conceptual coherence at different hierarchical levels of inductive reasoning, which suggests that the mental manipulation of relevant concepts in doing a task could be grouped into simple, complex, and an unconnected category. Researchers have focused on the exploration of the inductive inference and its psychological processes within conceptual hierarchies by examining the relationships between knowledge of concepts at different hierarchical levels, expectations about conceptual coherence, and inference. This observation is relevant to what Biggs and Collis (as cited in Moseley et al., 2005) have proposed in their Structure of the Observed Learning Outcome (SOLO) Taxonomy: that knowledge indeed has levels and structure.

Mathematical reasoning is an essential tool in mathematics learning. Scholars in the field acknowledge its importance and vast application for the modern world. Hatfield (1993) affirmed this by stressing that mathematical reasoning will help students to understand mathematics and its applications to everyday life. Ban Har (2007) similarly emphasized the importance of mathematical ability to the global economy. However, there is evidence of the failure of mathematical tasks in their application to real-world situations and multi-step problems. Many of these difficulties are due to the conceptualization gap. For example, when problems involve several pieces of information, or lack sufficient information to solve, students perform poorly (Cangelosi, 1992). Moreover, data from the Trends in International Mathematics and Science Study revealed 19 countries performing significantly below the international average for the reasoning domain (Mullis, Martin, & Foy, 2005). Low performance in this area is evident globally.

Neo-Piagetian research that characterizes prospective mathematics teachers' level and developmental trends of cognition in inductive reasoning is scant. The researchers of this study believe that an understanding of the cognition levels of inductive reasoning and the current level of students' conceptual facilities can help teachers design developmentally appropriate instructional material and classroom activities for learning geometric concepts using the inductive approach to teaching. We further believe that when a mismatch exists between students' priori knowledge and classroom instruction, or corresponding materials for developing a geometric notion, learning is inhibited. This study attempts to characterize students' cognition level when performing inductive geometry tasks using basic geometric concepts such as undefined terms and polygons. Specifically, this study examines students' cognition level and developmental trend in inductive reasoning.

Conceptual Framework

The SOLO Taxonomy

The SOLO Taxonomy describes a hierarchy of increasing complexities that learners exhibit in the mastery of academic tasks such as reasoning, understanding, and problem-solving. Students' responses to the academic task can fall into one of the five levels of cognition. Biggs and Collis (as cited in Mosley et al., 2005) identified these levels as *pre-structural, uni-structural, multi-structural, relational*, and *extended abstract*. These stages outline the structural organization of knowledge from incompetent to expert. Each subsequent level of the SOLO taxonomy demands an increasing amount of working memory or attention span. At the higher levels, situations are more complex with more features to consider, more relationships between parts to examine, and more distinctions to make between actual and hypothetical situations. The different levels are described as follows:

Pre-structural. At this level the students show partial understanding as reflected in their unconnected response processes. The responses generally miss the point.

Uni-structural. The responses can be correct but inconsistent. The learners can discuss content meaningfully as a reasonable amount of content is known. However, they cannot easily apply or transfer their knowledge. Learners can demonstrate concrete, reductive understanding of the topic and can make simple and obvious connections, but the broader significance is not understood. Important attributes are often missed.

Multi-structural. Learners understand several components of the task but remain inconsistent. They can connect many attributes but understanding of the entire problem is vague. Many ideas and concepts remain unrelated and disorganized. Learners may recognize two ideas or concepts but will fail to make any connection between them. The end result is often an incorrect conclusion.

Relational. Learners integrate numerous elements that show how bits of information are interconnected to each other. Responses are still anchored in concrete experiences and are often inconsistent, but learners' overall understanding allows for the application of concepts or ideas to a familiar task or work-based situation.

Extended abstract. Responses include all relevant data. The interconnectedness between ideas or concepts is understood and learners are able to hypothesize and theorize. Learners can deduce information that is not part of the original. The mind is not confined to a fixed conclusion and considers alternative solutions. At this level, a learner can transfer or apply learning to other fields of study.

The different levels are further discussed by Asquith (n.d.), Biggs (1996), Hattie, Biggs and Purdie (1996), Moseley et al. (2005), and Zachariades, Christou, and Papageorgiou (2000-2001). The University of South Australia also provides an informative description of the SOLO Taxonomy (University of South Australia, 2011).

Method

Research Design and Sampling

The study participants were purposively selected based on their academic standing. The academic standing was the criterion for the selection in order to verify the SOLO taxonomy. The study is based on the assumption that knowledge gained through formal schooling will have structure. It was expected that participants, due to their superior academic standing, would be able to better demonstrate their geometry knowledge. Thirty students were invited from three universities in Cebu City. The top ten math majors at each university were selected. Of the 30 selected, 27 successfully participated in the study.

Research Site and Participants

The three institutions were a state university, a religiously affiliated university, and a private university. Student and institutional participation was approved by the Dean of the College of Education at each of the respective institutions. The researchers coordinated with the chairman of the mathematics department to identify the top ten, third-year prospective teachers among the math major students. Students' performance and achievement in plane geometry was also a considered criterion for selection. The rationale for selecting the top ten is based on the assumption that these students will have a more advanced understanding of the concepts of geometry for cognition in inductive reasoning. The chairman arranged the day of the test administration with the consent of the participants.

Instrument

The test contained 24 tasks. Twelve of the tasks (1,3,5,8,14,21,23,10,12,18,19,24) included four sub-problems that were used for checking the prospective teachers' inductive reasoning skills. The 24 tasks were taken from different sources and were modified. A 5-item checklist, designed and based on the SOLO cognition level taxonomy, was utilized to elicit students' perceptions of their degree of exposure to the inductive reasoning tasks. All instruments were in English. The checklist was approved by one of the creators of the SOLO taxonomy, Professor John Biggs. The tasks and the checklist for the perception of exposure to inductive reasoning were validated by a university mathematics educator and by the Director of Research and Planning in Cebu Normal University. The checklist aided the researchers in understanding the prospective teachers' experiences of the inductive reasoning tasks.

Data Collection

The test was administered at different times during the day. The participants completed the set of inductive reasoning tasks in the morning and the deductive reasoning tasks in the afternoon. The participants were instructed by the researchers to show all possible solutions. To minimize test anxiety, there was no set time limit in answering the test questions. After completing the test each student filled out the checklist indicating their perception of exposure to reasoning tasks.

Data Processing Method

Three raters evaluated the solution processes of the participants. The first scorer was one of the researchers who had been teaching plane geometry for eight years. The second was a faculty member at the University of San Carlos-College who was completing her doctorate degree at the time of the study. She had taught plane geometry at Cebu Eastern College and served as the Mathematics and Science Coordinator for five years. The last assessor was the Director of Research and Planning at Cebu Normal University with expertise in research and evaluation. The evaluators independently assessed the students' answers.

The evaluators were guided by the following scoring guidelines: If a student engaged in any sub-problem but was not successful or left any sub-problem unanswered, the student was given a zero. An accurate answer for a sub-problem was given a point. For example, if sub-problems *a* and *b* were completed correctly, two points were given. If sub-problems *a*, *b*, and, *c* were done correctly then three points were given. An accurate response to sub-problems *a* through *d* received four points. The scores achieved by the students were used to cluster them into percentile quartiles. Simple percentages were used to categorize the difficulty level of each task. An in-depth analysis of students' solution processes was conducted to qualitatively describe students' levels of cognition and developmental trends.

Results and Discussion

The Students' Cognition Level and Developmental Trend in Inductive Reasoning

The following procedures were done to determine a possible developmental trend in cognition. First, the success rate for each task was identified to determine the easiest and the hardest tasks. This was done to determine the level of difficulty in completing the task. The tasks were classified as easy, average, and difficult based on the percentage of correct responses. Second, the students were grouped according to percentile quartiles. Four classes were defined: class1 (n=6), low achievers; class 2 (n=7), below average achievers; class 3 (n=8), average achievers; and class 4 (n=6), above average achievers. Lastly, the successful answers for each class were summarized and analyzed for the associated specific trends of cognition level. Table 1 presents the tasks category based on the precise answer.

The students were instructed to complete each task by formulating a mathematical rule or conjecture. Table 1 shows that task 1 was easy for the students as 70% of the students answered it correctly while the remaining tasks were difficult for them. No student provided the correct response to tasks 14 and 18. The presence of difficulty levels in inductive reasoning tasks seems to reflect a developmental pattern and a possible clustering of students. Based on this assumption, the students were grouped into percentile quartiles. Table 2 presents the groupings of the students. It shows the task completed by 50% or more of the students in classes 2, 3, and 4. The data indicates that there is a developmental trend in students' ability to complete the assigned task because the successful completion of a task by more than 50% of the students in each class (except for class 1) was associated with a similar accomplishment by more than 50% of the students in each succeeding class.

Classes 1, 2, and 3 performed the same level task. However students in class 2 performed with higher success compared to students in class 1, and students in class 3 performed with greater facility compared to students in class 1 or class 2. The findings imply that more students

Table 1
Difficulty Level of the Inductive Tasks and the Percentage of Correct Response

Item no.	%	Classification
1	70.37	Easy
3	3.70	Difficult
5	7.41	Difficult
8	7.41	Difficult
10	29.63	Difficult
12	7.41	Difficult
14	0.00	Difficult
18	0.00	Difficult
19	7.41	Difficult
21	11.11	Difficult
23	3.70	Difficult
24	18.52	Difficult

Note. The number of participants is 27. An item is considered easy if 66.66%-100% of the participants provided the correct answer. It is average if 33.33%-66.65% of the participants successfully provide the answer. An item is considered difficult if less than 33.32 % of the participants provided the correct answer.

in class 3 (87%) were able to complete making the rule for task 1 compared to the other two groups. As previously stated, task 1 was an easy task for the students (70% were able to make the rule). However, many class 1 students were not able to generalize the rule for the task. Additionally, only 57% of the students in class 2 succeeded in the task. Meanwhile, all students in class 4 were able to succeed in generalizing the rule. It is worth noting that 50% of the students in class 4 were able to accomplish difficult tasks like 10 and 21. Class 4 students were more successful than the other three classes in conjecture or rulemaking from a pattern. The findings further suggest that the cognitive skills of class 4 students were better than that of their peers when completing the tasks.

We conclude that students whose cognition is far better than that of others have a higher rate of success compared to other students who achieve the same. Thus, one will not be successful in completing a higher level task unless one succeeds at the lower level tasks. The suggestion that those who were unable to complete a higher level task unless they could first perform a task at lower levels, indicates that the hypothesized levels in Table 2 generate a hierarchy of cognition in inductive reasoning. The findings support Coley et al. (2004) who proposed that there is no one basic level. Eventually, Coley and his team found diverse hierarchical levels in the inductive inference of adults who are advantaged in different conceptual functions. In our study the hypothesized levels and associated developmental characteristics conform to Biggs and Collis' (1982) *thinking levels*. The students' written solutions were examined and are used below to describe their thinking process.

Level 1. At this level students were able to illustrate and establish obvious connections within a task. However, the significance of the connections to complete the task was not used.

Table 2

Developmental Trend of Cognition Level in Inductive Reasoning

Levels	Classes of students and the number of correct responses in some tasks			
	1(n=6)	2(n=7)	3(n=8)	4(n=6)
1	(1) 33.33%	(1) 57.14%	(1) 87.50%	(1) 100%
	2	4	7	6
2	0.00%	0.00%	0.00%	(10) 50%
				3
				(21) 50%
				3

Note. There are 27 students who took the test. No students gave the correct responses for items 14 and 18. The number below the % designates the exact number of successful answer.

Thus, the students were able to meet a single or a few requirements but were distracted as they proceeded further. For instance, students at this level were able to provide a rule correctly that there are six angles formed when there are two points in the interior part of an angle. However, they were unable to continue the investigation as the number of points in the interior part of the angle increased. Similarly, in task 8 they were able to indicate that four collinear points can form a maximum of six line segments but were not able to solve the problem with five collinear points. Moreover, students operating at this level were able to make the rule that there are three maximum number of intersection points of three coplanar points but were not able to do the same with four points.

Task 1 was the only task for which many of the students provided a rule suggesting that they lacked the knowledge of how to investigate the other tasks that involved inductive types of thinking. Kemp and Jern (2013) stated that inductive reasoning is critically dependent on background knowledge. Class 1 (n=6) students often exhibited level 1 developmental characteristics for task 1 where approximately 66% of class 1 students were not successful in providing the rule despite the fact that task 1 was easy for the majority (70%) of the participants. Also no students from class 1, 2, or 3 were able to provide a rule for task 8. The challenge revealed their difficulty with processing the information and in utilizing and integrating the concept of points and lines into the task. Likewise, no students from class 1, 2, or 3 provided a rule for tasks 3, 19, and 21, and therefore implying a similar level of difficulty.

Ashcraft (1994) inferred that the inability of a person to conceptualize and generalize is either due to the limited understanding of the information, or the incapability to use and integrate previously learned concepts into new problems. For example, approximately 92% of the students were not able to construct the rule for task 5 because they failed to use the concept of intersection points and lines throughout the investigation, thus demonstrating their inability to communicate and apply their understanding of intersection points of two lines in this particular context. Furthermore, this also implies that a lack of ability to apply learning to various contexts frustrated them, and as a result, they eventually stopped investigating. Data in Figure 1 illustrates the students' thinking process.

In Figure 1, sub-problem a in task 5 (the task required to generate the rule for the maximum number of intersection points p when there are intersecting n number of coplanar lines l) was

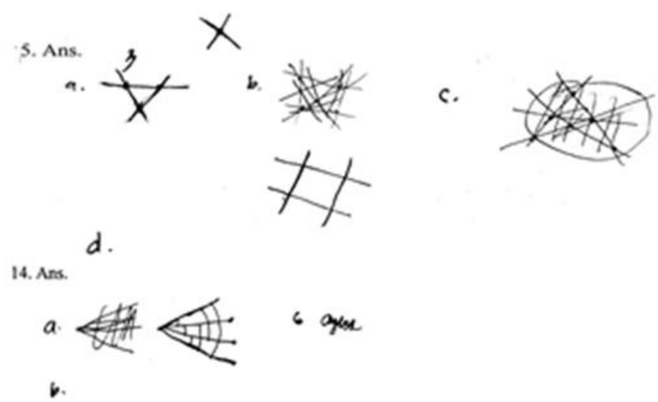

Figure 1. Collated examples of students' investigation for items 1 and 14 at cognition level 1. The students were not able to proceed to investigate the subtasks b, c, and d.

answered correctly. This is because sub-problem *a* is very obvious since the situation presented a condition that four intersection points are not possible with three lines. It suggests that three is the maximum number of intersection points of three coplanar lines. However, when the given number of lines is four, the students did not investigate further because they were apparently not able to visualize the possible intersections of four lines. The students erased the illustration for sub-problems *b* and *c* which indicated their inability to translate more complex information into coherent figures. The solutions presented for task 14 reflected a similar kind of difficulty.

Task 14 required students to identify the number of angles formed when there are *n* number of points in the interior part however only sub-problem *a* for the task was accomplished. These situations are indicative of a lower cognition level (level 1) that was identified by the three evaluators because for both tasks students provided the correct response for only sub-problem *a*. The students might have been successful with the other sub-problems in tasks 5 and 14 if the rule had already been known and mastered by the student. Students were expected to have this knowledge because they are majoring in mathematics, and geometry courses are part of the curriculum. Moreover, participants reported that they have often encountered similar tasks in pattern generation. Level 1 appeared to be a period of cognitive development characterized by the students' naïve and often undirected attempts to conceptualize a general rule. Their thinking was more revealing of what Biggs and Collis (as cited in Mosely et al., 2005) described as the uni-structural level in the sense that only one obvious answer is grasped.

Level 2. In contrast to level 1, students demonstrating level 2 thinking further investigated the task. The characteristic of this level was the students' improved ability to reason out inductively by meeting only a few task requirements. Students with this kind of cognition level in inductive reasoning identified further that there is a maximum of six intersection points given four coplanar lines. Likewise, students with cognition level 2 investigated further and determined that 10 line segments can be named given five collinear points. Similarly, students operating at this level identified that 15 angles, that are formed with four points, are in the interior part of the angle. The students also precisely determined that a possibility of six lines can be drawn from four distinct coplanar points. Moreover, students at this level recognized more than one relevant feature during the investigative process and attempted to apply knowledge to the task. For instance, students noted that the sums of the measures of the interior angles of a five and six sided polygon are 540° and 720° respectively. They investigated the polygons by multiplying the number of triangles formed when the diagonals are drawn from one vertex to another by 180°. Students assessed at cognition level 2 in inductive reasoning appeared to exhibit multi-structural characteristics.

The students demonstrating level 2 thinking further conceptualized the requirement for sub-problems *b* of task 5 and successfully provided the correct answer. Students at level 2 thinking were not confused in providing what was necessary for the sub-problem because they understood what was needed for the task at this point. However, as the students continued to sub-problem *c*, they were confused and distracted for they could not relate the answer from sub-problems *a* and *b* because sub-problem *c* created an impression that it was different from the previous two sub-problems. The condition that was in the question, “the maximum number of intersection points of five coplanar lines is ten,” appeared to distract students’ cognition. The students treated the condition as the basis to answer the follow-up question. This eventually created confusion and inconsistency in their thinking process. As reflected in Figure 2, the students stopped illustrating after answering sub-problem *b*.

This situation revealed the incapacity of the students to grasp the significance of what was achieved in sub-problems *a* and *b*. Hence, the three raters evaluate their cognition at level 2. The solution processes in Figure 2 that depicted the same type of thinking further support the raters’ evaluation. For example, students were able to provide the answers for sub-tasks *a* and *b* that asked for the maximum number of lines that can be constructed from three and four distinct coplanar points respectively. However, this was considered not significant by the students to the entire task because cognition was distracted by the condition that was presented in sub-problem *c*, “if five distinct coplanar points can determine a maximum of 10 lines.” The students may have considered the condition irrelevant to the preceding two sub-problems of task 21. The condition again created the impression that it is the basis for answering the follow-up question, “how

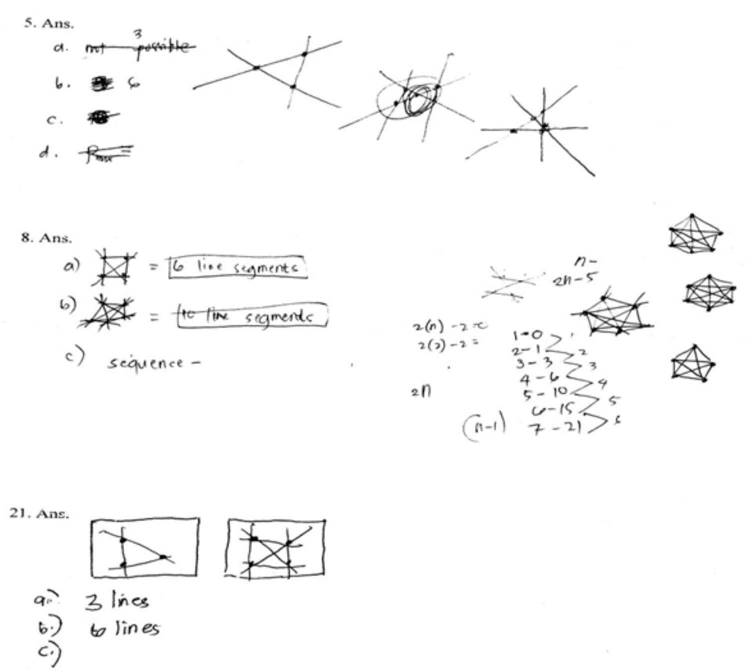

Figure 2. Examples of students’ investigations for items 5, 8, and 21 at cognition level 2. The students successfully answered subtask b.

many lines do six distinct collinear points determine?” It was again assumed by the students to be a different situation and not significant to the entire task. The students’ erroneous assumptions are a manifestation of the inconsistency of thought and a lack of understanding.

The student did not continue exemplifying their thought. Figure 2 presents that the thinking process in task 8 was not relevant. The item involved collinearity of points and the maximum of line segments that can be established. The students were able to meet the first two requirements; however, the thinking was contextually inconsistent because the illustration was not fitted to the tasks and the figure drawn did not depict collinearity of points, but rather it depicted distinct coplanar points.

Level 2 thinking appeared to constitute partial understanding where only a few requirements needed to generate a rule or to make a conjecture were met. However, the significance of the entire task as part of the general idea of the situation and its meta-connections was taken for granted. As noted by Klauer and Phye (2008), and discussed by Coley et al. (2004), conceptual coherence and specific knowledge is imperative in inductive inference. The level 2 thinking inferred that students’ conceptual understanding was incoherent and that there existed an apparent knowledge gap of the entire inductive tasks.

Conclusion

The present study supports two cognition levels of inductive reasoning ability (level 1 and level 2) that are compatible with the lower SOLO levels of complexities: the uni-structural and the multi-structural. The presence of the two levels of inductive reasoning ability in performing geometry inductive task items by the 27 prospective math teachers supports claims in the literature that human cognition has levels. The quality of the cognition depends on the quality of knowledge at hand. The two lower levels in the current study suggest two things: Students may have superficial knowledge of geometry concepts, and they are inefficient in transferring what is already available to them to apply it to the new task. The participants were expected to have good background knowledge and to perform well since they were among the top ten math majors at their respective universities. The participants’ inability to draw on their knowledge in inductive reasoning task may have been attributed to the inconsistency of exposure as evident by the students’ self-reports which indicated that students were exposed to this kind of thinking activity irregularly. The fact that some students performed only at level 1 or level 2, while others accomplished both the easy and the more complex tasks, suggests a trend in the development of their knowledge. Based on the findings, the researchers believe that those who cannot achieve a simple task become frustrated when performing the various higher sub-problems in each task. Typically, a student who has this capacity is not ready to grasp the relationship between and among the information in all the sub-problems as part of the entire inductive reasoning task.

Our hypothesis extends to the belief that there are other lower levels as projected by the incomplete tasks and those that were not engaged. Likewise, a higher level that characterizes thinking may also exist and be compatible with the SOLO levels. It is claimed, based on the evidence of this analysis, that the quality of knowledge is important as it governs taught processes, like for example, connecting and communicating the knowledge acquired to newly available information to complete the task. The findings support the literature cited in this study.

The findings of this study have implications for teaching and learning mathematics. In the classroom there are those that are not equipped with the knowledge needed to mediate what is planned by the teacher to achieve. This has to be addressed first by examining carefully the available knowledge the learners have. In doing so, teachers will have insight into designing instruction that is developmentally appropriate for the students. The authors suggest the SOLO taxonomy for assessing students' learning capacity, and then devising activities and instructional materials that scaffold learning experiences that develop a concept from simple into an integrated set of complex knowledge.

The low cognition level reported in this study presents a dismal scenario of the quality of future mathematics educators. It is hoped our study will provide guidance to the curriculum developer regarding the design of intervention programs so that the performance in inductive reasoning can be raised. Mathematics curriculum at all fields should include competency skills in conjecture and inference. The teacher as facilitator of learning must have sound pedagogical skill to enhance this mathematical thinking activity. It is recommended that students be exposed to such activity regularly. The training of the mind, to see a pattern and the generalization that governs it, is essential to mathematics learning. The role of the teacher is to provide learners with rich classroom experiences that involve knowledge construction, conjecture, and rulemaking.

The sample size of this study is relatively small and is limited only to those who outperformed their peers, a study employing a similar research design but using a large and more diverse set of learners is recommended.

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