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Research Note

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Reconceptualizing the Mathematical Preparation of Secondary School Mathematics Teachers

In this research note, we offer theoretical reflections underlying a current ongoing research program on the mathematical education of secondary school mathematics teachers, illustrating thereby its directions and innovative character.

The Context and its Issues

Currently in Canada, as well as in other countries, secondary teachers are commonly required to take a significant number of academic courses at the university level in their subject specialization, for example, mathematics, in order to teach at the secondary school level. One issue that has received recent attention in the mathematics education literature concerns the divide between the mathematical experiences teachers encounter in these courses and the practice of teaching mathematics in schools. In effect, mathematical content knowledge, as a fundamental component in most mathematics teacher education programs, is usually identified with formal academic mathematics (which often are courses offered for future mathematicians). Although these courses may be important for future mathematicians, research studies are currently questioning whether these courses are of value to mathematics teachers. By identifying teachers' mathematical content knowledge with academic mathematics, the subject matter preparation for mathematics schoolteachers is often conceived as a self-contained process, implicitly promoting values, approaches, concepts, and ways of thinking appropriate to academic mathematics, but not necessarily appropriate to the practice of teaching mathematics in schools. This orientation has significant implications for the constitution of schoolteachers' professional ways of knowing and practicing mathematics.

Academic Mathematics: Issues of Content, Form, and Practices

Research studies have shown that the emphasis on the formal nature of the mathematics in most academic mathematics courses may have the detrimental effect of reinforcing the abstract and technical aspects of mathematics in teach-

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ers' understanding of concepts as well as in their teaching (Ball, Lubienski, & Mewborn, 2001; Cooney & Wiegel, 2003; Gattuso, 2000). This can lead to serious difficulties in how teachers make mathematics comprehensible to students (Nathan & Koedinger, 2000; National Research Council, 2001; Thompson & Thompson, 1994; Thompson & Thompson, 1996;). Although one strength of academic mathematics is to compress mathematical ideas so that they are more powerful and easier to use, the opposite appears to be relevant for teaching school mathematics to students (Adler & Davis, 2006; Ball & Bass, 2003; Moreira & David, 2005). In order to foster students' mathematical understandings, teachers must be able to unpack, dismantle, and decompress mathematical concepts to allow the meanings and subtleties hidden in their compact structure to emerge. Mathematics teaching practices require a return to the underlying meanings of concepts in order to promote robust mathematical comprehension in students (Bednarz, 2001; Brousseau, 1998; Ma, 1999); this is an emphasis outside the focus of academic mathematics, thus augmenting the disconnection toward teachers' teaching practices. Through insistence on formalism and abstraction, studies in academic mathematics do not focus on developing the knowledge that teachers will use in their professional teaching practices.

Another important issue is how these academic mathematics courses are taught: primarily through modes of lecturing and the exposition of mathematical knowledge (Bauersfeld, 1998; Burton, 2004). The habits and ways of doing developed in these courses are, therefore, more about standardized knowledge than about participation in a process of learning that reflects teachers' classroom practices. According to Bauersfeld, teachers need to be immersed in a practice of doing mathematics, a culture of mathematics, rather than merely being introduced to a body of objective knowledge (where mathematics is an epistemological absolute). Participating in mathematical practices is to enter into practice that uses mathematics; that shares and negotiates its meaning; that generates ideas, questions, norms, and ways of doing in mathematics: a practice where mathematics is created and alive.

Research on Teachers' Knowledge of School Mathematics: Rethinking Mathematical Experiences

Simultaneously, research points to important difficulties that secondary teachers experience with aspects of school mathematics. For example, studies by Ball (1990) and Bryan (1999) have illustrated that although the secondary mathematics teachers whom they studied made few if any mistakes in their use of mathematical procedures, they experienced significant difficulties in providing sound meaning and explanations for the mathematical rationales underlying these procedures. Other studies have highlighted difficulties of another order concerning secondary teachers' unfamiliarity with the meaning of concepts themselves and solving processes (e.g., definitions, conjectures, relationships within concepts). For example, Even (1993) and Hitt-Espinosa (1998) observed that many teachers possessed an "old" definition of a function as a continuous graph, preventing them from recognizing or accepting alternative drawings as representing a function and leading them to transform or treat discrete functions as continuous. Also, Schmidt and Bednarz (1997) and Van Dooren,

Verschaffel, and Onghena (2003) reported on secondary teachers' difficulties in appreciating arithmetic procedures as valid solutions to traditional algebra problems. Although these types of studies have been criticized as presenting a deficit model of teacher knowledge, and moreover cannot be generalized to all teachers, they offer significant information about the mathematical experiences that could be offered in teacher education initiatives. These studies demonstrate a need to provide teachers with opportunities to explore and enrich their knowledge of school mathematics concepts and suggest teacher education practices that emphasize mathematical aspects/concepts closely related to their professional practices. These issues underpin the objectives of our research program.

Objectives and Orientations of the Research Program

These theoretical reflections form the basis of our current research program, which is invested in creating and studying approaches to professional development (PD) for secondary mathematics schoolteachers. These PD initiatives attempt to offer teachers rich experiences in exploring school mathematics and immerse them in an engaging mathematical practice. Our work focuses on analyzing the mathematical knowledge, understandings, and even teaching practices that teachers develop through these initiatives. These intentions are encapsulated in the following three axes.

1. To study the development of secondary teachers' mathematical knowledge and practices: What sorts of mathematical ways of knowing do teachers develop through these teacher education practices? How do they evolve through these experiences in relation to their knowledge, ways of doing, and of engaging with mathematics?
2. To study the interrelationship between teachers' development of mathematical ways of knowing and their practices of teaching: How do teachers interpret and make sense of their experiences in these teacher education initiatives, as mathematical doers and as teachers? How, if at all, are these experiences reinvested in their teaching practices in schools?
3. To reinform teacher education practices: How do these results shed light on teacher education practices?

Concluding Remarks

These reflections suggest an important shift concerning the mathematical learning opportunities offered to teachers in mathematics teacher education practices. We are well aware, as various colleagues have expressed to us, that these orientations appear provocative as well as counterintuitive because they question well-established, accepted structures of teacher education as well as the disciplinary content of academic mathematics as suitable knowledge for schoolteachers. It is, however, through these alternative experiences that we believe that teachers will have the opportunity to continue growing in mathematics and enhancing their ways of professionally knowing, practicing, and teaching mathematics. As more attention is given to the importance of the mathematical preparation of mathematics teachers, our study participates in, and will help stimulate, current national and international discussions about

the orientations and potential *reconceptualizations* of mathematics teacher education practices.

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