John A. Ross

Douglas McDougall

and

Anne Hogaboam-Gray

Ontario Institute for Studies of Education, University of Toronto

Research on Reform in Mathematics Education, 1993-2000[1]

Proponents and opponents of reform of mathematics education all cite the research base in support of their positions. This article reports the results of a review of studies that contained empirical evidence of the effects of reform or the difficulty of implementing reform that were published between 1993 and 2000. The studies reviewed indicate that implementation of math reform contributes to student achievement, but evidence abounds of superficial implementation and barriers to enactment. There are well-documented strategies for reducing these barriers, the most promising strategies being inservice that simultaneously focuses on teachers' practice and their cognition about mathematics teaching.

Promoteurs et adversaires d'une réforme en enseignement des mathématiques puisent tous dans la recherche pour appuyer leurs points de vue. Cet article présente le résultat d'une analyse d'études empiriques publiées entre 1993 et 2000 et qui évoquent les effets d'une réforme ou la difficulté d'en mettre une en application. Selon les études analysées, la mise en oeuvre d'une réforme en enseignement des mathématiques contribue aux réalisations des élèves. Toutefois, énormément d'articles évoquent une mise en oeuvre superficielle et des obstacles à la réforme. Parmi les stratégies bien documentées pour minimiser ces obstacles, celles qui promettent le plus impliquent une formation pour enseignants en cours d'emploi qui touche à la fois les méthodes des enseignants et leurs connaissances sur l'enseignement des mathématiques.

When the California State Board of Education sought guidance from researchers about mathematics teaching and learning, E.D. Hirsch, Jr. provided an unambiguous answer: Research says that "only through intelligently directed and repeated practice, leading to fast, automatic recall of math facts, and facility in computation and algebraic manipulation can one do well at real-world problem solving" (Becker & Jacob, 2000, p. 535). The Board greeted Hirsch's summary with a standing ovation and included it in their rationale for dismantling reform initiatives enacted by their predecessors.

John A. Ross is a professor in the Department of Curriculum, Teaching, and Learning and head of the Institute's field center in Peterborough, Ontario. His research interests are mathematics education, student assessment, and program evaluation.
Anne Hogaboam-Gray is the research officer at the field center in Peterborough. Her research interests are qualitative and quantitative data analysis procedures and student assessment.
Douglas McDougall is an assistant professor in the Department of Curriculum, Teaching, and Learning. His research interests are mathematics education, use of technology in education, and change in mathematics teachers.

In this article we provide a different account of what research says by reviewing empirical studies of mathematics teaching reported in academic journals and conferences between 1993 and 2000. After describing our search procedures we define math education reform, summarize the results of the review, and describe implications for educators and researchers. The main argument of the review is that the California State Board asked the wrong question. The issue is not whether reform in mathematics teaching contributes to student achievement (it does), but why implementation has been such a rare, fleeting occurrence and what can be done to support teachers' efforts to change their practice.

Search Procedures

The review was the first step in a school improvement effort funded by the Ontario Ministry of Education that focused on grades 7-8 mathematics teaching (Impact Math). The central intervention strategy was the design and delivery of an inservice program to volunteer teachers in school districts across the province. The purpose of the review was to ground the inservice in empirical evidence by compiling answers from current research to three questions: (a) Does the implementation of reform in mathematics education contribute to improved student achievement? (b) What are the barriers to implementing reform? (c) How can these barriers be overcome? We were sympathetic to the ideals of Standards-based reform (described below) and had previously conducted a number of research projects on strategies for implementing it.

Our immediate goal was to conduct a literature review that was systematic, reproducible, and explicit (Fink, 1998). We used a combination of manual and machine searches. We began by manually searching mathematics journals, general educational research journals that publish studies of mathematics learning, and academic conferences. The manual search identified keywords[2] that were used in ERIC searches. The database was expanded through a final manual search (i.e., references cited by studies caught in the initial search).

We used three criteria to select studies for the review. First, the study had to contain empirical evidence, either quantitative or qualitative, of the effects of enacting education reform or data on implementation processes. We excluded reports that described but did not assess instruction, prescriptions for practice based solely on intuition and experience, policy statements unsupported by evidence, and theory development articles in which no original data were collected. Second, the study had to contain an overt strategy by which some aspect of reform was implemented; that is, one or more of the 10 dimensions listed below (adapted from Ross, Hogaboam-Gray, McDougall, & Bruce, 2001-2002). This excluded studies focusing on student or teacher characteristics that affect outcomes, unless the student or teacher attribute was included as a moderator of a treatment. Third, the search was limited to studies reported between 1993 and 2000. Our rationale was that the latest round of reform began with the publication of the National Council of Teachers of Mathematics (NCTM) Standards in 1989. We estimated that that it took several years before the Standards were incorporated into field studies and subjected to rigorous review. One hundred, thirty-four studies met the inclusion criteria (listed in Ross, 2000); this set was expanded by 20 studies added in response to suggestions by reviewers.

Coding Studies

Studies were coded in terms of sample (size, grade, student, and teacher demographics), theoretical framework, methodology (including instructional treatment and measurement instruments), results, and implications for Impact Math.[3] In coding study quality we were particularly concerned to avoid errors observed in earlier research syntheses, such as unexplained selectivity, author misrepresentations of findings, and unwarranted attributions by the reviewer of study conclusions (Dunkin, 1996; Guglielmi & Tattrow, 1998; Matt & Cook, 1994). Rather than discarding studies that were flawed, we coded design quality as a study attribute that increased or reduced confidence in the findings. We constructed a rubric, shown in Table 1, that contained three levels based on credibility (for qualitative designs) and internal validity (for quantitative designs). Five coders independently reviewed an initial sample of five studies. Differences in interpretation of code categories were resolved through discussion. The remaining studies were coded by a single reviewer (60% by the lead author).

We created a summary of each study organized around the coding categories. These summaries were used to create a narrative review organized around the three study questions. We opted for a narrative research synthesis rather than a quantitative meta-analysis, because we did not wish to exclude studies that lacked statistical information required for the calculation of effect sizes. In making methodological decisions we were guided by the principles for reviewing qualitative studies developed by Schreiber, Crooks, and Stern (1997).

Characteristics of Reform

Reform in mathematics education is motivated by the finding that traditional teaching has produced low performance on basic competence tests (Romberg, 1997); the recognition that the world into which students will graduate requires greater ability to use mathematical tools (Bossé, 1995; Heid, 1997); and by

Table 1
Rubric for Judging Study Quality

Criteria	*Level 1*	*Level 2*	*Level 3*
Publication	Not refereed	Peer reviewed conference paper	Peer reviewed journal article
Quantitative Design	Obvious flaws, e.g., nonequivalent groups	Minor problems, e.g., nonequivalent groups with statistical adjustment	Few problems, e.g., equivalent groups or statistical controls
Qualitative Design	No overt credibility procedures*	2-3 overt credibility procedures*	4+ overt credibility procedures*

*Credible qualitative designs include one or more of the following techniques: triangulation of data sources (i.e., compare data using different instruments or subjects); triangulation over time (i.e., compare data collected at different times); triangulation of observers (i.e., compare data collection by different observers); triangulation of interpretations (i.e., ask others to interpret data); member checks, accurate recording (e.g., audiorecording); maintaining an audit trail (tracking themes from raw data); and rich description.

advances in pedagogy that emphasize building on student prior knowledge, peer learning, and knowledge construction (Fennema, Franke, & Carpenter, 1993). No single set of attributes characterizes all reform initiatives, but we can identify central tendencies that distinguish traditional from reform approaches.

The chief characteristics of math education reform that emerge from the review and NCTM policy statements (1989, 1991, 2000) are as follows.

1. Broader scope (e.g., multiple math strands with increased attention on those less commonly taught such as probability rather than an exclusive focus on numeration and operations).
2. All students have access to all forms of mathematics, including teaching complex mathematical ideas to less able students.
3. Student tasks are complex, open-ended problems embedded in real-life contexts; many of these problems do not afford a single solution. In traditional math, students work on routine applications of basic operations in decontextualized, single solution problems. Leighton, Rogers, and Maguire (1999) suggested that formal (traditional) tasks differ from informal (reform) tasks in that formal tasks hold all relevant information required to solve the problem (whereas informal tasks require the solver to bring knowledge to the problem), are self-contained, provide a single correct answer, can be solved using conventional procedures, involve solutions that are unambiguous, entail topics that are of academic interest only, and do not prepare students to solve real-life problems.
4. Instruction in reform classes focuses on the construction of mathematical ideas through students' talk rather than transmission through presentation, practice, feedback, and remediation.
5. The teacher's role in reform settings is that of co-learner and creator of a mathematical community rather than sole knowledge expert.
6. Mathematical problems are undertaken in reform classes with the aid of manipulatives and with ready access to mathematical tools (calculators and computers), support not present in traditional programs.
7. In reform teaching, the classroom is organized to encourage student-student interaction as a key learning mechanism rather than to discourage it as an off-task distraction.
8. Assessment in the reform class is authentic (i.e., analogous to tasks undertaken by professional mathematicians), integrated with everyday events, and taps a wide variety of abilities, in contrast with end-of-week and unit tests of near transfer that characterize assessment in traditional programs.
9. The teacher's conception of mathematics in the reform class is that of a dynamic (i.e., changing) discipline rather than a fixed body of knowledge.
10. Teachers in reform settings make the development of student self-confidence in mathematics as important as achievement (Pajares, 1996, reported evidence that mathematics self-efficacy in the junior grades was a better predictor of senior math achievement than junior math achievement).

These elements can be found in provincial guidelines. For example, the intended curriculum in Ontario (Ontario Ministry of Education and Training, 1997) includes all 10 of these features, while omitting some elements often included in reform initiatives such as having students invent algorithms (Ball, 1993; Carroll, 1996). The Ontario curriculum also includes a feature not usually associated with math reform: detailed grade-level expectations.

This list of reform characteristics is not an unorganized set of disembodied teaching behaviors. The dimensions overlap and constitute an orientation to instruction that differs fundamentally from traditional practice. Each teacher enacts these dimensions in unique ways. Yet there are patterns of excellence across teachers that make it possible to talk about a central tendency of reform teaching that is consistent in the subject and congruent with reform in other subjects (Sternberg & Horvath, 1995). For example, Borko and Putnam (1995) suggested that expert teachers have a cognitive mediational view of learning (i.e., that students relate incoming information to existing knowledge, impose meaning on experience, and monitor their learning processes) that translates into a constructivist approach to teaching. Although expertise in teaching in a given subject shares characteristics of excellence in teaching other subjects, subject-specific enactments differ, as demonstrated by studies that examine the effects of out-of-subject assignments on teacher practice and cognition about that practice (Ross, Cousins, Gadalla, & Hannay, 1999).

Research on the Effects of Reform on Students and Teachers

Research on math reform from 1993 to 2000 took two paths. The first consisted of a relatively small number of studies of the effects of reform on student achievement. When reform was implemented these effects were positive. Some of the most convincing evidence comes from qualitative studies that tracked teachers over several years as they elicited rich talk as students solved rich, meaningful problems in mathematical communities created in their classrooms. For example, Fennema et al. (1993) tracked a teacher over four years as she implemented Cognitively Guided Instruction, a program that focused on helping students construct deep understanding of mathematical concepts and strategies for solving problems embedded in their everyday experiences. Fennema et al. found that this exemplary teacher with a deep understanding of the structure of mathematics and children's mathematical thinking had a profound effect on her grade 1 students. They solved more complex problems than other grade 1 pupils, used higher-level strategies, and adapted their procedures in response to problem requirements. They were knowledgeable about what they knew, had positive affect for the subject, persisted in problem-solving when confronted by obstacles, and were fluent in describing their thinking.

Boaler (1993, 1994, 1997, 1998) conducted an extensive longitudinal study of two schools in the United Kingdom, tracking students from age 12 to 16. In Phoenix, a school characterized by a commitment to math education reform, students worked in cooperative groups on three-week projects, asked their teacher when they wanted input on math concepts (i.e., concepts were only introduced when needed), and classroom talk emphasized construction of student thinking. In contrast in the other school, Amber Hill, the program emphasized individual workbooks and textbooks. Classrooms were characterized by a search for correct answers rather than understanding, competition, individual work, and the transmission of algorithms and procedures. When given open-ended tasks, Phoenix students outperformed students in Amber Hill. Phoenix students were willing to derive meaning from the problem, and they were able to select an appropriate procedure or adapt one to fit a new situation. In contrast, the knowledge of Amber Hill students was inert: they could not apply their knowledge. Boaler concluded that in Phoenix students learned how to use their knowledge. Phoenix students performed more consistently (i.e., they tended to use intuitive methods on all problems) and were enabled rather than distracted by contextual features. In contrast, Amber Hill students were negatively influenced by superficial problem features and used standard school algorithms regardless of their appropriateness: they were unable to transfer their knowledge. Boaler also noted that students' attitudes toward mathematics were consistently better in Phoenix, especially for girls, who also enjoyed a reduction in the gender achievement gap.

These rigorously conducted qualitative studies are persuasive because of the readers' knowledge of the effects of traditional mathematics programs. There is overwhelming evidence that such programs lead to mastery of basic algorithms without conceptual understanding. As Hiebert (1999) notes, old math is a proven failure. When we encounter evidence that Standards-based programs promote deep understanding, it has an inter-ocular effect: it hits you between the eyes.

Several quantitative studies report that classrooms that provide rich tasks embedded in the real-life experiences of children, with rich discourse about mathematical ideas, and a focus on children's thinking contribute to deeper understanding. For example, Cardelle-Elawar (1995) found that such a program contributed to superior grades 3-8 student performance on a problem-solving measure. Brenner et al. (1997) reported that a program with similar features for grades 7-8 had positive effects on problem-solving outcomes valued by reformers (such as the ability to represent mathematical relationships in multiple ways). The Core Plus Math Project that embodied Standards principles in curriculum materials increased secondary school students' skills emphasized in reform agenda such as interpreting charts and tables and promoted deep understanding of algebra and geometry concepts (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Schoen, Fey, Hirsch, & Coxford, 1999). Silver and Stein (1996) found that a program that provided high-level problems to students produced growth in students' understanding, reasoning, and problem-solving. Gains were larger in classes that implemented the tasks as intended (Stein, Gover, & Henningsen, 1996). In contrast, Ganter (1994) found that that providing rich tasks to college students and structuring collaborative development of solutions had no effect on conceptual understanding, although there was a positive effect on students' attitudes to mathematics.

Villasenor and Kepner (1993) found that children in classrooms that fully implemented math reform were also more successful on traditional math tasks, a finding reflected in international comparisons that report higher achievement in countries that have adopted reform practices such as Japan (Reys, Reys, & Koyama, 1996; Romberg, 1997). Schoen et al. (1999) also found a significant advantage for Core Plus Math students on a traditional algebra test, although the advantage did not endure beyond the first year of the study. Brenner et al. (1997) found no effects of Standards-based programming on a measure of traditional math objectives. Mayer (1998) found that on a traditional, multiple-choice algebra test, Standards-based programs had a positive effect, but only for higher-ability students. Although the results of these studies are mixed, with some reporting no significant advantage for Standards-based programs, no studies show such programs producing results lower than those attained through traditional mathematics instruction.

There is also evidence that reform contributes to the achievement of disadvantaged students (Silver & Stein, 1996), as well as those of average ability, although there is limited evidence that lower-status students might be disadvantaged in reform classrooms (Lubienski, 2000). In summary, students in classrooms characterized by mathematics education reform have higher achievement on achievement measures emphasized by reformers such as problem-solving and conceptual understanding, have more positive attitudes toward the subject, and at least have no less achievement on objectives emphasized by traditional programs such as computational efficiency.[4]

Math reform is difficult to implement, however. The second set of studies, larger than the first, focused on evidence of nonimplementation and barriers to enactment. Even teachers chosen as exemplars of reform practice regress from the ideal, displaying the height of reform one day, but regressing to traditional methods the next (Senger, 1998). Some elements of reform are more difficult to implement than others. The most challenging is the management of students' talk about mathematical reasoning—including finding the right balance between encouraging student constructions without leaving them floundering (Ball, 1993; Ross & Cousins, 1995a, 1995b; Ross, Haimes, & Hogaboam-Gray, 1996; Smith, 2000; Williams & Baxter, 1996).

The catalogue of barriers to reform is lengthy. Among the most important are: teachers must be agents of a change they did not experience as students (Anderson & Piazza, 1996). The pedagogy is not only different, but also harder to learn. For example, in traditional math there is a generic script that guides each day's lesson through a manageable body of content. In reform math the day is governed by unpredictable student responses to real-life problems. Teachers, especially elementary generalists, tend to lack the disciplinary knowledge required to make full use of rich problems (Henningsen & Stein, 1997; Lloyd & Wilson, 1998; Mandeville & Liu, 1997; Monk, 1994; Phillip, Flores, Sowder, & Schappelle, 1994; Spillane, 2000; Stein et al., 1996) and texts cannot prescribe universally applicable courses of action (Remillard, 2000). Adoption of reform math can leave teachers feeling less efficacious because their contribution to student learning is less visible than in traditional teaching (Ross, McKeiver, & Hogaboam-Gray, 1997; Smith, 1996). Teachers' beliefs about mathematics (i.e., a rigid set of algorithms, not understandable by most students, that must be approached in an inflexible sequence) conflict with reform conceptions of math as a fluid, dynamic set of conceptual tools that can be used by all (Gregg, 1995; Prawat & Jennings, 1997). Reform does not meet parental expectations about how math should be taught and tested (Graue & Smith, 1996; Lehrer & Shumow, 1997). Reform conceptions of mathematics conflict with mandated assessment programs that measure computational speed and accuracy (Firestone, Mayrowetz, & Fairman, 1998). Time to cover the curriculum is a major challenge. Keiser and Lambdin (1996) found that students' constructions took longer than lecture-recitations, novel problems increased time taken for discussion of homework, and students with poor motor skills took longer than anticipated to use manipulatives.

Reducing Barriers to Implementation

There have been many suggestions for increasing classroom applications of reform ideals (e.g., policy development, preservice training, materials development, alignment of assessment with instruction, etc.).

The most powerful method for increasing implementation is inservice. In reviewing similarities between the current round of math reforms and the New Math movement of the 1950-1960s that ultimately failed to influence teacher practice, Bossé (1995) noted that inattention to teacher inservice was the key deficiency of both movements. The Standards of Practice (NCTM, 1989) anticipated that teachers would be able to develop materials and practices to enact the vision of reform with little support. Experience since then demonstrates that it is essential to provide ongoing professional development, particularly focused on providing teachers with examples of constructivist teaching (Bitter & Hatfield, 1994) and explicitly addressing their beliefs about mathematics as a teachable subject (Grant, Peterson, & Shojgreen-Downer, 1996). The delivery sequence (i.e., changes in beliefs before adjustment of practices or vice versa) appears not to matter: what is essential is that inservice contain both components (Borko, Davinroy, Bliem, & Cumbo, 2000). Especially important is public and private reflection. Sharing professional experiences is such an essential element of professional growth that it has become axiomatic that inservice events should provide opportunities for participants to describe their experiences, reflect on the meanings of personal practice, and exchange interpretations with colleagues (Fullan & Connelly, 1990; Grimmett & Erickson, 1988; Kemessis, 1987). Evidence of the positive effects of such inservice on teacher implementation of math education reform and student achievement is accumulating (Knapp & Peterson, 1995; Pligge, Kent, & Spence, 2000; Schifter & Simon, 1992; Smith, 2000). There is also evidence that provision of new curriculum materials in the absence of sustained inservice has little impact on teacher implementation (Price, Ball, & Luks, 1995; Roulet, 1998).

Another promising approach to reducing barriers to implementation of reform in mathematics education is through integration with technology. There is ample correlational evidence that teachers who are more frequent users of technology (calculators, computers) are more likely to adopt even the most difficult dimensions of reform such as constructivist teaching (Becker, 1998; Waxman & Huang, 1996). Provision of software in a reform curriculum contributes to teacher implementation of the Standards (Huetinck, Munshin, & Murray-Ward, 1995; Ross, Hogaboam-Gray, & McDougall, 2000). Student achievement increases when calculators (Hembree & Dessart, 1992) and computers (Christmann, Badgett, & Lucking, 1997; Heid, 1997) are used.

Less clear about the integration of computers is how this contributes to reform. The relationship may be spurious: good teachers tend to adopt the innovations of the day, in this case technology integration and math reform (Becker, 1998). It is more likely that technology enables teachers to implement their constructivist beliefs by relieving students of the tedium of calculation and providing them with visual representations to support dialogue about mathematical ideas. Some researchers (Sandholtz, Ringstaff, & Dwyer, 1997) have argued that technology demands that teachers change to a constructivist orientation because they have to share control with students in a computer-based learning environment. The contribution of technology to math reform is not automatic. Providing computers and software to teachers without appropriate inservice has minimal effect on their practice (Robertson et al., 1996).

A less promising strategy, frequently advocated, is curriculum alignment: of assessment and curriculum or curriculum integration across subjects. It is argued that schools will improve if governments set clear standards for students and teachers, assess the extent to which standards are met using curriculum aligned tests, and provide schools with feedback (Teddlie & Reynolds, 2000). The credibility of the argument is threatened when raw test scores are used. The only Canadian study to report the effect of school population factors on provincial test scores (Lytton & Pyryt, 1998) found that none of the variance in grade 3 and grade 6 mathematics scores could be attributed to instructional factors. Mandated testing programs provide some support for reform in mathematics teaching when the tests reflect reform learning goals. State tests influence teachers' choice of content, although not their instructional strategies (Firestone et al., 1998) and have only a modest impact on achievement (Shepard et al., 1996). Positive effects have been observed when mathematics and science curricula have been aligned around problem-based units (Austin, Hirstein, & Walen, 1997; Ross & Hogaboam-Gray, 1998) or around the structure of mathematics (Isaacs, Wagreich, & Gartzman, 1997).

Several studies investigated the effect of restructuring on implementation of reform mathematics. Changes in teaching strategy to improve student-student communication and female achievement in mathematics have been reported when single-sex classes or schools were created (Parker & Rennie, 1997), but most of these studies were methodologically flawed (Mael, 1998). Teachers' knowledge and use of reform practices increased when schools established partnerships with outside agencies, particularly the NCTM (Mills & Garet, 1996, found that membership of the department head in the NCTM increased reform implementation); universities that provide inservice on reform practices (Borko, 1997; Brahier, 1998; Ross, 1995a); and school networks (Hernandez-Gantes & Brendefur, 1997). Site-based management, in which instructional decision making at the school level is shared with teachers, also contributes to curricular change (Wagstaff, 1995). Each of these studies identified restructuring as a key change element. A more likely explanation is that restructuring stimulated collaboration among teachers, which led to instructional innovation. Ross et al. (1997) found that teacher collaboration contributed to implementation of mathematics education reform. The least experienced teacher in this qualitative study benefited the most from collaboration, because it reduced her workload, clarified expectations for content coverage, and set the pace of instruction. Other teachers benefited from peer emotional support, identification of new teaching strategies, and workload sharing. The contribution of collaboration to implementation of math reform was observed by in other studies (Feikes, 1998; Moreira & Noss, 1995; Ponte, Matos, Guimarães, Leal, & Canavarro, 1994).

A large number of investigations reported during the time of the review addressed new ways of teaching mathematics in the reform classroom. Most of these instructional improvement approaches displayed a high degree of rigor, although the degree of reform implementation could not always be determined. For example, several studies focused on demonstrating that cooperative learning techniques contribute to achievement of reform ideals (Kiczek & Maher, 1998; Mulryan, 1995; Slavin & Madden, 1999; Whicker, Bol, & Nunnery, 1997). However, some researchers found evidence in mixed-ability groups of passivity on the part of less able students in mathematical discussions and dysfunctional responses to their learning needs on the part of higher-ability students (King, 1993; Ross, 1995a, 1995b). A major theme in this literature was the search for ways to make cooperative groups more effective. Some studies focused on grouping strategies, finding that in mathematics class, homogeneous ability grouping is preferable for complex problems (Fuchs, Fuchs, Hamlett, & Karns, 1998), but only if students have different bodies of knowledge to draw on to solve problems (Mevarech & Kremarski, 1997). Other studies focused on procedures for improving the quality of discourse in student groups. Researchers found that training students how to give explanations had a positive effect on mathematics achievement (Fuchs et al., 1997; Hoek, van den Eeden, & Terwel, 1999; Nattiv, 1994), especially when the training was focused specifically on how to give mathematical explanations (Fuchs et al.).

Implications of Research on Mathematics Education Reform

Research reported in 1993-2000 found, first, that reform in mathematics education contributed to higher student achievement. Although the number of studies that have investigated achievement effects is relatively small, the studies reviewed in the achievement section of this article were of high quality, with all of them reaching at least level 2 in the quality rubric provided in Table 1 and the majority reaching level 3. These positive results were attained only when there was substantial implementation of reform, a rare event. There was evidence of unintended variation in treatments in districts, schools, and teachers, and some reform elements were more difficult to implement than others. The key implication for math educators is to recognize that despite the outbreak of the "mathematics wars" in many countries, the research base encourages teachers and schools to implement the Standards. We found no evidence that warrants a regression to past practices as implied by the Hirsch quotation at the beginning of the article. The research also indicates that progress toward implementing reform ideals will be incremental, with advances occurring on a broken front with many backward steps.

Measuring teacher change will be problematic. There is no consistent image of what reform should look like in the classroom and even less consensus about how it should be measured. The Standards are an accumulation of the visions of its writers. Although the philosophy behind the Standards is appropriately described as constructivist, this was a label assigned by Romberg to gain political support after the Standards were written (Bossé, 1995). It is unclear how the dimensions of reform should be weighted. Although there have been attempts to describe levels of implementation on particular reform dimensions (Bright, Bowman, & Vace, 1998; Franke, Carpenter, Levi, & Fennema, 2001; Gabriele et al., 1999; Hall, Alquist, Hendrickson, & George, 1999; Lambdin & Preston, 1995; McDougall et al., 2000; Nolder & Johnson, 1995; Ross et al., 2000;

Ross, Hogaboam-Gray, & McDougall, 2001; Slavit, 1996; Spillane & Zeuli, 1999) and suggestions for conceptual tools (such as discourse analysis) for distinguishing levels of classroom practice (Blanton, Berenson, & Norwood, 2001), no overall rubric has been created that has broad approval. Researchers need to provide: (a) a rubric for guiding the generation of instruments, (b) a self-report survey for tracking the progress of large groups of teachers, and (c) coding schemes for observing classrooms for use in qualitative studies.

Second, the studies provided consistent evidence of the barriers to reform. Although the quality of the studies reviewed in this section of the article was more variable, study quality was randomly distributed over the topics reviewed (i.e., none of the claims made in this section was based on a corpus of studies that was so consistently flawed as to introduce systemic bias). The most important obstacle is that teachers' beliefs and prior experiences of mathematics and mathematics teaching are not congruent with the assumptions of the Standards. Teachers mostly support the goals of reform, but overestimate the extent to which their practices approach these goals. The innovation is ambiguous and difficult to implement. The lack of accessible examples impedes the development of local visions, implementation is costly in terms of classroom and teacher preparation time, substantive change in practice threatens teachers' beliefs about their efficacy, and the complexity of student tasks is prone to diminution. Realization of reform ideals is also thwarted by policy misalignments, the most important being competition with other innovations and conflicts with mandated student assessment programs. The key implication for reformers is to encourage modesty in expectations about impact and to anticipate widespread variation in the use of reform ideas.

Third, the research identified promising strategies for overcoming barriers to reform. The most powerful mechanism is professional development. Sustained interaction is needed between classroom teachers with professional development leaders external to the school together with provision of local support such as district or school consultants. There needs to be a dual emphasis on new classroom strategies while attending to teachers' cognition about their existing practice. There should also be a dual focus on developing teachers' disciplinary knowledge (knowing the subject) and their pedagogical content knowledge (i.e., knowing how to present mathematical content to students and being able to anticipate and respond to students' misconceptions about the material to be learned). In making the argument for a professional development focus, we are mindful that inservice alone is insufficient to bring about teacher change, although research to date indicates it is the most powerful agent of change. Schoenfield (2001), for example, suggests four other essential conditions: quality curriculum aligned with the Standards; a stable, knowledgeable, and professional teaching community; quality assessment aligned with Standards; and a fine balance between stability and mechanisms for evolution.

Finally, a variety of successful instructional experiments have been reported. The most promising involve strategies for teaching students how to talk about mathematics in cooperative learning settings. As experiments flourish, researchers in partnership with teachers will begin to realize the promise of reform by addressing the unresolved teaching issues identified by

Gutstein and Romberg (1995), such as how algorithms can be taught in a meaningful way while maintaining a commitment to students' inventions.

Notes

1. Preparation of this review was funded by the Ontario Ministry of Education and Training. Ann Kjander and Alex Lawson reviewed some of the studies. An earlier version of the review is available in Ross, Hogaboam-Gray, & McDougall (2000).
2. The keywords in the first search, focused on implementation issues, were: mathematics with educational change, educational innovation, professional development, program implementation, reform efforts, and large scale programs. In the second search, focused on effects of reform on achievement, the keywords were: mathematics with achievement, education, instruction, and skills.
3. We also coded the studies in terms of intended outcomes (categorized in terms of the four criteria in the provincial mathematics rubric) and domain of mathematics (the five strands specified in provincial curriculum guidelines) but insufficient information was provided in most studies so these codes were not used.
4. All the studies reviewed in this section received high scores on the rubric for study quality. A few studies with less rigorous procedures produced comparable results. For example, Simon and Schifter (1993) found that students exposed to a Standards-based program had deeper understanding, greater facility in communicating mathematical ideas, and more positive attitudes to the subject, but there were no gains on standardized test scores. However, this study was level 1 quality. It provided no reliability or validity information on the measures used, data from different standardized tests were pooled, there was a heavy reliance on teacher self-reports, and it employed a pre-post cohort design without controls. The study did not report descriptive data such as means or statistical procedures, used grade-equivalent scores in statistics rather than raw scores, and analyzed each survey item separately without Bonferonni adjustments for multiple comparisons.

References

Anderson, D.S., & Piazza, J.A. (1996). Changing beliefs: Teaching and learning mathematics in constructivist preservice classrooms. *Action in Teacher Education, 17*(2), 51-62.

Austin, J., Hirstein, J., & Walen, S. (1997). Integrated mathematics interfaced with science. *School Science and Mathematics, 97*(1), 45-49.

Ball, D.L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary mathematics. *Elementary School Journal, 93*, 373-397.

Becker, H. (1998, April). *The influence of computer and Internet use on teachers' pedagogical practices and perceptions*. Paper presented at the annual meeting of the American Educational Research Association, San Diego.

Becker, J.P., & Jacob, B. (2000). The politics of California School Mathematics. *Phi Delta Kappan, 81*, 529-537.

Bitter, G.G., & Hatfield, M.M. (1994). Training elementary mathematics teachers using interactive multimedia. *Educational Studies in Mathematics, 26*, 405-409.

Blanton, M.L., Berenson, S.B., & Norwood, K.S. (2001). Using classroom discourse to understand a prospective mathematics teacher's developing practice. *Teaching and Teacher Education, 17*, 227-242.

Boaler, J. (1993). Encouraging the transfer of "school" mathematics to the "real world" through the integration of process and content, context and culture. *Educational Studies in Mathematics, 25*, 314-373.

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to "realistic" mathematic contexts. *British Educational Research Journal, 20*, 551-564.

Boaler, J. (1997). Reclaiming school mathematics: The girls fight back. *Gender and Education, 9*, 285-305.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*(1), 41-62.

Borko, H. (1997). New forms of classroom assessment: Implications for staff development. *Theory into Practice, 36*, 231-238.

Borko, H., Davinroy, K.H., Bliem, C.L., & Cumbo, K.B. (2000). Exploring and supporting teacher change: Two third-grade teachers' experience in a mathematics and literacy staff development project. *Elementary School Journal, 100*, 273-306.

Borko, H., & Putnam, R. (1995). Expanding a teacher's knowledge base: A cognitive psychological perspective on professional development. In T. Guskey & M. Huberman (Eds.), *Professional development in education: New paradigms and practices* (pp. 35-65). New York: Teachers College Press.

Bossé, M.J. (1995). The NCTM Standards in light of the New Math movement: A warning! *Journal of Mathematical Behavior, 14*, 171-201.

Brahier, D.J. (1998). The effects of staff development focused on assessment practices. *Proceedings of the International Group for the Psychology of Mathematics Education, North America, 20*, 199-204.

Brenner, M.E., Mayer, R.E., Moseley, B., Brar, T., Duran, R., Reed, B.S., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal, 34*, 663-689.

Bright, G.W., Bowman, A.H., & Vace, N.N. (1998). Reflections about listening and dialogue as evidence of teacher change. *Proceedings of the International Group for the Psychology of Mathematics Education, North America, 20*, 607-612.

Cardelle-Elawar, M. (1995). Effects of metacognitive instruction on low achievers in mathematics problems. *Teaching and Teacher Education, 11*(1), 81-95.

Carroll, W.M. (1996). Use of invented algorithms by second graders in a reform mathematics curriculum. *Journal of Mathematical Behavior, 15*, 137-150.

Christmann, E., Badgett, J., & Lucking, R. (1997). Microcomputer-based computer-assisted instruction within differing subject areas: A statistical deduction. *Journal of Educational Computing Research, 16*, 281-296.

Dunkin, M.J. (1996). Types of errors in synthesizing research in education. *Review of Educational Research, 66*(2), 87-97.

Feikes, D. (1998). The role of formal and informal support networks as teachers attempt to change their practice from traditional to innovative reform-oriented teaching of mathematics. *Proceedings of the International Group for the Psychology of Mathematics Education, North America, 20*, 749-753.

Fennema, E., Franke, M., & Carpenter, T. (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal, 30*, 555-583.

Fink, A. (1998). *Conducting research literature reviews: From paper to the Internet.* Thousand Oaks, CA: Sage.

Firestone, W.A., Mayrowetz, D., & Fairman, J. (1998). Performance-based assessment and instructional change: The effects of testing in Maine and Maryland. *Educational Evaluation and Policy Analysis, 20*(2), 95-113.

Franke, M.L., Carpenter, T.P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal, 38*, 653-690.

Fuchs, L., Fuchs, D., Hamlett, C., Phillips, N., Karns, K., & Dutka, S. (1997). Enhancing students' helping behavior during peer-mediated instruction with conceptual mathematical explanations. *Elementary School Journal, 97*, 223-250.

Fuchs, L.S., Fuchs, D., Hamlett, C.L., & Karns, K. (1998). High-achieving students interactions and performance on complex mathematical tasks as a function of homogeneous and heterogeneous pairings. *American Educational Research Journal, 35*, 227-267.

Fuchs, L.S., Fuchs, D., Karns, K., Hamlett, C., Katzaroff, M., & Dutka, S. (1997). Effects of task-focused goals on low-achieving students with and without learning disabilities. *American Educational Research Journal, 34*, 513-543.

Fullan, M.G., & Connelly, M. (1990). *Teacher education in Ontario: Current practice and options for the future.* Toronto, ON: Ministry of Colleges and Universities.

Gabriele, A., Joram, E., Trafton, P., Thiessen, D., Rathmell, E., & Leutzinger, L. (1999, April). *Traveling along the path of mathematics reform: Changes in teachers' source of efficacy.* Paper presented at the annual meeting of the American Educational Research Associated, Montreal.

Ganter, S.L. (1994). The importance of empirical observations of mathematics programs: A case from the calculus reform movement. *Focus on Learning Problems in Mathematics, 16*(2), 1-19.

Grant, S.G., Peterson, P.L., & Shojgreen-Downer, A. (1996). Learning to teach mathematics in the context of systemic reform. *American Educational Research Journal, 33*, 509-541.

Graue, M.E., & Smith, S.Z. (1996). Parents and mathematics education reform. *Urban Education, 30*, 395-421.

Gregg, J. (1995). The tensions and contradictions of the school mathematics traditions. *Journal for Research in Mathematics Education, 26*, 442-466.

Grimmett, P., & Erickson, G. (1988). *Reflection in teacher education.* New York: Teachers College Press.

Guglielmi, R.S., & Tatrow, K. (1998). Occupational stress, burnout, and health in teachers: A methodological and theoretical analysis. *Review of Research in Education, 68*, 61-99.
Gutstein, E., & Romberg, T. (1995). Teaching children to add and subtract. *Journal of Mathematical Behavior, 14*, 283-324.
Hall, G.E., Alquist, A., Hendrickson, M., & George, A.A. (1999, April). *Bringing national standards for the teaching and learning of mathematics to the classroom: Implementation, student outcomes and system impacts*. Paper presented at the annual meeting of the American Educational Research Association, Montreal.
Heid, M.K. (1997). The technological revolution and the reform of mathematics. *American Journal of Education, 106*, 5-61.
Hembree, R., & Dessart, D.J. (1992). Research on calculators in mathematics education. In J.T. Fey & C.R. Hirsch (Eds.), *Calculators in mathematics education* (pp. 23-32). Reston, VA: National Council of Teachers of Mathematics.
Henningsen, M., & Stein, M.K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*, 524-549.
Hernandez-Gantes, V.M., & Brendefur, J.L. (1997, March). *Producing authentic, integrated, Standards-based mathematics curriculum: [More than just] a collaborative approach*. Paper presented at the annual meeting of the American Educational Research Association, Chicago.
Hiebert, J. (1999). Relationships between research and the NCTM Standards. *Journal for Research in Mathematics Education, 31*, 3-19.
Hoek, D., van den Eeden, P., & Terwel, J. (1999). The effects of integrated social and cognitive strategy instruction on the mathematics achievement in secondary school. *Learning and Instruction, 9*, 427-448.
Huetinck, L., Munshin, S., & Murray-Ward, M. (1995). Eight methods to evaluate support reform in the secondary-level mathematics classroom. *Evaluation Review, 19*, 646-662.
Huntley, M.A., Rasmussen, C.L., Villarubi, R.S., Sangtong, J., & Fey, J.T. (2000). Effects of Standards-based mathematics education: A study of the Core-Plus Mathematics Project algebra and functions strand. *Journal of Research in Mathematics Education, 31*, 328-361.
Isaacs, A., Wagreich, P., & Gartzman, M. (1997). The quest for integration: School mathematics and science. *American Journal of Education, 106*, 179-206.
Keiser, J.M., & Lambdin, D.V. (1996). The clock is ticking: Time constraint issues in mathematics teaching reform. *Journal of Educational Research, 90*, 23-31.
Kemessis, S. (1987). Critical reflection. In M. Wideen & I. Andrews (Eds.), *Staff development for school improvement: A focus on the teacher* (pp. 73-90). New York: Falmer Press.
Kiczek, R.D., & Maher, C.A. (1998). Tracing the origins and extensions of mathematical ideas. *Proceedings of the International Group for the Psychology of Mathematics Education, North America, 20*, 377-382.
King, L. (1993). High and low achievers' perceptions and cooperative learning in two small groups. *Elementary School Journal, 93*, 399-416.
Knapp, N.F., & Peterson, P.L. (1995). Teachers' interpretations of "CGI" after four years: Meanings and practices. *Journal for Research in Mathematics Education, 26*, 40-65.
Lambdin, D.V., & Preston, R.V. (1995). Caricatures in innovation: Teacher adaptation to an investigation-oriented middle school mathematics curriculum. *Journal of Teacher Education, 46*, 130-140.
Lehrer, R., & Shumow, L. (1997). Aligning the construction zones of parents and teachers for mathematics reform. *Cognition and Instruction, 15*(1), 41-83.
Leighton, J.P., Rogers, W.T., & Maguire, T.O. (1999). Assessment of student problem solving on ill-defined tasks. *Alberta Journal of Educational Research, 45, 409-427.*
Lloyd, G.M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29*, 248-274.
Lubienski, S.T. (2000). A clash of social class cultures? Students' experiences in a discussion-intensive seventh-grade mathematics program. *Elementary School Journal, 100*, 377-403.
Lytton, H., & Pyryt, M. (1998). Predictors of achievement in basic skills: A Canadian effective schools study. *Canadian Journal of Education, 23*, 281-301.
Mael, F.A. (1998). Single-sex and coeducational schooling: Relationships to socioemotional and academic discipline. *Review of Educational Research, 68*(2), 101-129.
Mandeville, G.K., & Liu, Q. (1997). The effect of teacher certification and task level on mathematics achievement. *Teaching and Teacher Education, 13*, 397-407.

Matt, G.E., & Cook, T.D. (1994). Threats to validity of research synthesis. In H. Cooper & L. Hedges (Eds.), *The handbook of research synthesis* (pp. 503-520). New York: Russell Sage Foundation.

Mayer, D.P. (1998). Do new teaching standards undermine performance on old tests? *Educational Evaluation and Policy Analysis, 20*(2), 53-73.

McDougall, D., Lawson, A., Ross, J.A., MacLellan, J., Kajander, A., & Scane, J. (2000). *A study on the Impact Math implementation strategy for the Ontario Mathematics Curriculum, Grades 7 & 8*. Toronto, ON: OISE/UT.

Mevarech, Z.R., & Kremarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal, 34*, 365-394.

Mills, V.L., & Garet, M.S. (1996). *When resources matter: Supporting departmental features which facilitate reform in the teaching of high school mathematics*. Paper presented at the American Educational Research Association, New York.

Monk, D.H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), 125-145.

Moreira, C., & Noss, R. (1995). Understanding teachers' attitudes to change in a logomathematics environment. *Educational Studies in Mathematics, 28*, 155-176.

Mulryan, C. (1995). Fifth and sixth graders' involvement and participation in cooperative small groups in mathematics. *Elementary School Journal, 95*, 297-310.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics: An overview*. Reston, VA: Author.

Nattiv, A. (1994). Helping behaviors and math achievement gain of students using cooperative learning. *Elementary School Journal, 94*, 285-297.

Nolder, R., & Johnson, D. (1995). Professional development: Bringing teachers to the center of the stage. *Mathematics in School, 24*(1), 32-36.

Ontario Ministry of Education and Training. (1997). *Grades 1-8 mathematics*. Toronto, ON: Author.

Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Research, 66*, 543-578.

Parker, L.H., & Rennie, L.J. (1997). Teachers' perceptions of the implementation of single-sex classes in coeducational schools. *Australian Journal of Education, 41*(2), 119-133.

Phillip, R.A., Flores, A., Sowder, J.T., & Schapelle, B.P. (1994). Conceptions and practices of extraordinary mathematics teachers. *Journal of Mathematical Behavior, 13*, 155-180.

Pligge, M.A., Kent, L.B., & Spence, M.S. (2000). *Examining teacher change within the context of mathematics curriculum reform: Views from middle school teachers*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Ponte, J.P., Matos, J.F., Guimarães, H.M., Leal, L.C., & Canavarro, A.P. (1994). Teachers' and students' views and attitudes towards a new mathematics curriculum: A case study. *Educational Studies in Mathematics, 26*, 347-365.

Prawat, R., & Jennings, N. (1997). Students as context in mathematics reform: The story of two upper-elementary teachers. *Elementary School Journal, 97*, 251-270.

Price, J., Ball, D., & Luks, S. (1995). *Marshalling resources for reform: District administrators and the case of mathematics research*. East Lansing, MI: Michigan State University.

Remillard, J. (2000). Can curriculum materials support teachers' learning? Two fourth-grade teachers' use of a new textbook. *Elementary School Journal, 100*, 331-350.

Reys, B., Reys, R., & Koyama, M. (1996). The development of computation in three Japanese primary-grad textbooks. *Elementary School Journal, 96*, 423-438.

Robertson, S.I., Calder, J., Fung, P., Jones, A., O'Shea, T., & Lambrechts, G. (1996). Pupils, teachers and palmtop computers. *Journal of Computer Assisted Learning, 12*, 194-204.

Romberg, T.A. (1997). The influence of programs from other countries on the school mathematics reform curricula in the United States. *American Journal of Education, 106*, 127-147.

Ross, J.A. (1995a). Students explaining solutions in student-directed groups: Cooperative learning and reform in mathematics education. *School Science and Mathematics, 95*, 411-416.

Ross, J.A. (1995b). Effects of feedback on student behavior in cooperative learning groups in a grade 7 math class. *Elementary School Journal, 96*, 125-143.

Ross, J.A. (2000). Full research review. In D. McDougall, A. Lawson, J. Ross, J. MacLellan, A. Kajander, & J. Scane (Eds.), *A study on the Impact Math implementation strategy for the Ontario mathematics curriculum, grades 7 & 8* (pp. 57-96) Toronto, ON: OISE/UT.

Ross, J.A., & Cousins, J.B. (1995a). Giving and receiving explanations in cooperative learning groups. *Alberta Journal of Educational Research, 41*, 104-122.

Ross, J.A., & Cousins, J.B. (1995b). Impact of explanation seeking on student achievement and attitudes. *Journal of Educational Research, 89*, 109-117.

Ross, J.A., Cousins, J.B., Gadalla, T., & Hannay, L. (1999). Administrative assignment of teachers in restructuring secondary schools: The effect of out-of-field course responsibility on teacher efficacy. *Educational Administration Quarterly, 35*, 782-804.

Ross, J.A., Haimes, D., & Hogaboam-Gray, A. (1996). Improving student helpfulness in cooperative learning groups. *Journal of Classroom Interaction, 31*(2), 13-24.

Ross, J.A. & Hogaboam-Gray (1998). Integrating mathematics, science, and technology: Effects on students. *International Journal of Science Education, 20*, 1119-1135.

Ross, J.A., Hogaboam-Gray, A., & McDougall, D. (2000). *Elementary school reform and higher order reasoning in mathematics*. Final report of MET Transfer Grant.

Ross, J.A., Hogaboam-Gray, A., & McDougall, D. (2001). *Measuring the effects of elementary school reform in mathematics*. Final report of MET Transfer Grant.

Ross, J.A., Hogaboam-Gray, A., McDougall, D., & Bruce, C. (2001-2002). The contribution of technology to the implementation of mathematics education reform: Case studies of grade 1-3 teachers. *Journal of Educational Computing Research, 26*, 123-140.

Ross, J.A., McKeiver, S., & Hogaboam-Gray, A. (1997). Fluctuations in teacher efficacy during the implementation of destreaming. *Canadian Journal of Education, 22*, 283-296.

Roulet, R.G. (1998). *Exemplary mathematics teachers: Subject conceptions and instructional practices*. Unpublished doctoral dissertation, University of Toronto.

Sandholtz, J.H., Ringstaff, C., & Dwyer, D. C. (1997). *Teaching with technology: Creating student-centered classrooms*. New York: Teachers College Press.

Schifter, D., & Simon, D. (1992). Assessing teachers' development of a constructivist view of mathematics learning. *Teaching and Teacher Education, 8*(2), 187-197.

Schoen, H.L., Fey, J.T., Hirsch, C.R., & Coxford, A.F. (1999). Issues and options in the math wars. *Phi Delta Kappan, 80*, 444-453.

Schoenfield, A.H. (2001). Making mathematics work for all children. *Educational Researcher, 31*(1), 13-25.

Schreiber, R., Crooks, D., & Stern, P. N. (1997). Qualitative meta-analysis. In J.M. Morse (Ed.), *Completing a qualitative project: Details and dialogue* (pp. 311-326). Thousand Oaks, CA: Sage.

Senger, E.S. (1998). Examining teacher and classroom supports for student invention. *Proceedings of the International Group for the Psychology of Mathematics Education, North America, 20*, 234-243.

Shepard, L.A., Flexer, R.J., Hiebert, E.H., Marion, S.F., Mayfield, V., & Weston, T.J. (1996). Effects of introducing classroom performance assessments on student learning. *Educational Measurement: Issues and Practice, 15*(3), 7-18.

Silver, E.A., & Stein, M.A. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*, 476-521.

Simon, M.A., & Schifter, D. (1993). Toward a constructivist perspective: The impact of a mathematics teacher inservice program on students. *Educational Studies in Mathematics, 25*, 331-340.

Slavin, R.E. and Madden, N.A. (1999, April). *Success for All/Roots and Wings: 1999 summary of research on achievement outcomes*. Paper presented at the annual meeting of the American Educational Research Associated, Montreal.

Slavit, D. (1996). Graphing calculators in a "hybrid" algebra II classroom. *For the Learning of Mathematics, 16*(1), 9-14.

Smith, J.P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*, 387-402.

Smith, M.S. (2000). Balancing old and new: An experienced middle teacher's learning in the context of mathematics instructional reform. *Elementary School Journal, 100*, 351-376.

Spillane, J.P. (2000). A fifth-grade teacher's reconstruction of mathematics and literacy teaching: Exploring interactions among identity, learning, and subject matter. *Elementary School Journal, 100*, 307-330.

Spillane, J.P., & Zeuli, J.S. (1999). Reform and teaching: Exploring patterns of practice in the context of national and state mathematics reforms. *Educational Evaluation and Policy Analysis, 21*(1), 1-27.

Stein, M., Gover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*, 455-488.

Sternberg, R., & Horvath, J. (1995). A prototype view of expert teaching. *Educational Researcher, 24*(6), 9-18.

Teddlie, C. & Reynolds, D. (Eds.). (2000). *The international handbook of school effectiveness research.* London: Falmer.

Villasenor, A., & Kepner, H.S. (1993). Arithmetic from a problem-solving perspective: An urban implementation. *Journal for Research in Mathematics Education, 24*, 62-69.

Wagstaff, J.G. (1995). Site-based management, shared decision making, and science and mathematics education: A tale of two districts. *Theory into Practice, 34*, 66-73.

Waxman, H.C., & Huang, S.Y. (1996). Classroom instruction differences by level of technology use in middle school mathematics. *Journal of Educational Computing Research, 14*, 157-169.

Whicker, K.M., Bol, L., & Nunnery, J.A. (1997). Cooperative learning in the secondary mathematics class. *Journal of Educational Research, 91*, 42-48.

Williams, S., & Baxter, J. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *Elementary School Journal, 97*, 21-38.