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Gender Differences in Growth in Mathematical Skills During Secondary Grades: A Growth Model Analysis

The purpose of this study was to examine differences in the rate at which males and females acquired mathematical skills during secondary grades and to determine the relationship between their rates of growth and student- and school-level characteristics. Based on data from the Longitudinal Study of American Youth (LSAY), a three-level hierarchical linear model (HLM) indicated that males and females acquired mathematical skills at the same rate. The variation in scores among students increased over time, and students maintained their initial position in the distribution. This fan-spread phenomenon among students was more pronounced in basic skills and knowledge than in problem-solving. Schools were more homogeneous in female rate of growth than in male rate of growth. The variation in scores among schools enlarged over time, and schools maintained their initial position in the distribution. This fan-spread phenomenon among schools was more pronounced in basic skills and knowledge than in problem-solving. Policy implications and suggestions for future research are provided.

Le but de cette recherche était d'étudier les différences dans le rythme auquel les adolescents et les adolescentes ont acquis des habiletés en mathématiques pendant leurs années au secondaire et d'établir le rapport entre leur taux de croissance et des caractéristiques liées aux individus d'une part et à l'école d'autre part. D'après les données du Longitudinal Study of American Youth (LSAY), un modèle linéaire à trois niveaux (HLM) a indiqué que les filles et les garçons acquéraient des habiletés en mathématiques au même rythme. La variation dans les résultats s'est accrue avec le temps et les élèves ont maintenu leur position relative dans la distribution. Ce phénomène d'éventail chez les élèves était plus prononcé pour les habiletés et les connaissances de base que pour la capacité de résolution de problèmes. Les écoles présentaient des profils plus homogènes pour le taux d'assimilation des habiletés chez les filles que chez les garçons. La variation dans les résultats par école s'est accrue avec les temps et les écoles ont maintenu leur position relative dans la distribution. Ce phénomène d'éventail parmi les écoles était plus prononcé pour les habiletés et les connaissances de base que pour la capacité de résolution de problèmes. On présente des implications en ce qui concerne la politique et des suggestions pour la recherche à l'avenir.

Before the mid-1980s a number of researchers found that differences between males and females in mathematics achievement were negligible during the elementary grades, noticeable during the intermediate grades, and pronounced during the high school grades (Burton et al., 1986; Crosswhite, Dossey, Swafford, McKnight, & Cooney, 1985; Ethington & Wolfle, 1984; Fennema, 1984; Leder, 1985; Peterson & Fennema, 1985). Researchers explained the gender gap in mathematics performance from biological and social psychological perspectives (Baker & Jones, 1992, for a review). The biological perspective

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attributed gender differences to the role of the X-chromosome, the differences in brain lateralization, and metabolic and hormone differences (Benbow & Stanley, 1983), whereas the social psychological perspective explained gender differences in the context of home, school, and community (Eccles & Jacobs, 1986; Fennema, 1985; Walkerdrine, 1988).

However, during the last decade, gender differences in mathematics have undergone dramatic changes, with two trends clearly emerging. One trend is a decline in the gap between mathematics achievement of males and that of females (American Association of University Women, 1992; Beller & Gafni, 1996; Manger, 1995; National Assessment of Educational Progress, 1997; Tartre & Fennema, 1995). Recent meta-analytic reviews show that gender differences in mathematics performance are either small (Friedman, 1996; Frost, Hyde, & Fennema, 1994) or declining over time (Friedman, 1989; Hyde, Fennema, & Lamon, 1990). The decline in the gender gap of mathematics achievement appears not only in mathematics as a whole, but also in various mathematical areas. For example, Ethington (1990) did not find substantial gender effects in mathematical areas such as fractions, ratio/proportion/percent, algebra, geometry, and measurement. Battista (1990) showed that males and females do not differ in their use of geometric problem-solving strategies. Although some studies continue to claim gender differences in mathematics (Mills, Ablard, & Stumpt, 1993; Randhawa & Randhawa, 1993), gender as a factor often seems to be less important than other psychological and sociological factors. For example, Byrnes and Takahiro (1993) found that although males outperform females on the SAT-Mathematics, gender explains no unique variance in mathematics achievement. They concluded that students' prior mathematical knowledge and problem-solving strategies matter more than students' gender.

The other trend is a substantial cross-regional (class, school, or nation) variation in both size and direction of gender differences in mathematics. For example, researchers have indicated that gender differences in mathematics achievement vary significantly across countries (Baker & Jones, 1993; Baker, Riordan, & Schaub, 1995; Hanna, 1989, 1990). Variation in gender differences is particularly apparent when class is taken as the unit of analysis. For example, Hart (1989) demonstrated that the differences between males and females in their interactions with the mathematics teacher vary considerably from classroom to classroom. Battista (1990) reported a significant teacher-by-student interaction in geometry and that "certain instructional practices may create or exacerbate these [gender] differences" (p. 59). Smith and Glynn (1990) found that mathematics teachers interact more with students of the same gender. Attitudes toward mathematics exhibited by females in single-gender mathematics classrooms and females in mixed-gender mathematics classrooms are quite different (Gwizdala & Steinback, 1990), and Mallam (1993) claimed that the best environment in which females learn mathematics appears to be in all-girls' schools where mathematics is taught by female teachers. Fennema (1981) noticed the variation in gender differences in mathematics among schools and concluded that

> The most important place to look to see if change is taking place is in schools themselves.... Some schools have been remarkably successful in helping females

learn mathematics and feel good about themselves as learners of mathematics. Other schools have not. (p. 93)

The cross-regional variation seems to support the social psychological perspective. Parental expectations (Eccles & Jacobs, 1986; Ramos, 1996), curricular and instructional materials (Boaler, 1994, 1997), the nature of teacher-student interactions (Leder, 1989), teacher attitudes and classroom processes (Jungwirth, 1991; Reyes & Stanic, 1988), mathematics course-taking patterns (Oakes, 1990), psychological factors such as attitude, motivation, and confidence (Fennema, Hyde, Ryan, & Frost, 1990; Ramos & Lambating, 1996; Terwilliger & Titus, 1995), educational environment (Burton, 1986; Campbell, 1995; Leder, 1986; Walden & Walkerdrine, 1986), school programs and educational policies (Durost, 1996; Jacobs & Wigfield, 1986), and local employment opportunities (Raffe & Willms, 1989; Willms & Kerr, 1987) all contribute to the unique pattern of gender differences in certain societies. Leder (1986) concluded that "a clear recognition of the values, expectations and beliefs of the wider society in which learning takes place is required for a full appreciation of the currently found gender differences in mathematics participation and performance" (p. 6).

Almost all studies on gender differences in mathematics achievement have been cross-sectional in scope, examining students' status rather than their rate of growth in mathematics achievement. Few researchers have taken the position that the "very notion of learning implies growth and change" (Willett, 1988, p. 346). Willms and Jacobsen (1990) examined the differences in the rates of growth in three domains of mathematical skills (computation, concepts, and problem-solving) between males and females during the intermediate years of schooling. They suggested that growth in all three domains (especially in the domain of concepts) is "fan-spread" during the intermediate years, indicating that the variation in scores increases over time and that students maintain their initial achievement position in the distribution. Willms and Jacobsen also found large differences between schools in their average rates of growth even after controlling for students' initial ability.

Willms and Jacobsen (1990) employed a sample of students from one Canadian school district. There has been no examination on the rate of growth in mathematics achievement from a national perspective (with a nationally representative sample). In addition, although Willms and Jacobsen (1990) discovered a large between-school variation in the rate of growth, they did not include any school-level variables in their study. Little is known about the relationship between school characteristics and the rate of growth in mathematics achievement. The present study was an attempt to fill these gaps. Data from the Longitudinal Study of American Youth (LSAY), a national five-year panel study (grades 7-11) of public middle and high school students in the United States with a focus on mathematics and science education (Miller & Hoffer, 1994) were analyzed. In addition to mathematics achievement, various student and school variables were also measured in the LSAY. Consequently, it was possible to include variables describing school composition and examine the effects of school composition on students' rate of growth in mathematics achievement in the present study.

In sum, a social psychological perspective was adopted in the present study to explain gender differences in the initial status (at the end of grade 7) and the rate of growth in mathematics. The main research questions were:

1. Do males and females differ in mathematics achievement at the end of grade 7? If so, are gender differences related to student background variables (age, socioeconomic status [SES], number of parents, and number of siblings) and school composition variables (school size, school location, and school mean SES)?
2. Does the rate of growth in mathematics achievement from grades 7 to 11 differ between males and females? If so, are gender differences related to student background and school composition variables?
3. Are there school differences between males and females in mathematics achievement at the end of grade 7 and in rate of growth in mathematics achievement? In other words, are schools different in promoting gender equalities?

Method

Data

Data for the present study were drawn from the stratified national probability sample of 52 middle and high schools in the Longitudinal Study of American Youth (LSAY) (Miller & Hoffer, 1994). The LSAY began in the fall of 1987, with samples of about 60 10th-graders, defined as *cohort 1*, and about 60 7th-graders, defined as *cohort 2* in each of 52 locations throughout the US. The 10th and 7th graders were followed for five years. The longitudinal data used in the present study came from cohort 2, covering the 1987-1988 school year (students were in grade 7) to the 1991-1992 school year (students were in grade 11). The total sample size was 3,116 students (1,490 females and 1,626 males).

Measures

The outcome variable was mathematics achievement. The LSAY used mathematics items from the National Assessment of Educational Progress (NAEP), selecting sets of items that demonstrated good measures of three theoretical dimensions as defined in the NAEP: (a) skill and knowledge, (b) routine application, and (c) problem-solving and understanding. Rearranging the items, the LSAY measured mathematics achievement in three skill dimensions: (a) basic skills and knowledge, (b) routine problem-solving, and (c) complex problem-solving. The present study used formula scores adjusted for difficulty, reliability, and guessing by means of item response theory (IRT) (Miller & Hoffer, 1994). As a result, test scores can be compared across test forms and grade levels.

Student background variables included gender, age, SES, number of parents, and number of siblings. Gender came from student self-reports, and this variable was checked against the students' first names and verified with each parent, with miscoding corrected. Based on the year born (1971 to 1979 exclusive of 1978), age was coded in unit increments from –4.5 to 4.5. Thus younger students in the sample were assigned negative values, whereas older students were assigned positive values. SES was a standardized composite variable of parents' self-reported education and occupation (modified Duncan scale) and student-reported household possessions. Marital status and the number of siblings were obtained from parent interviews. There were five categories of marital status: married, widowed, divorced, separated, and never married. Marital status was recoded to create a dummy variable, number of parents, with *married* as 1 and other categories as 0, based on the base-year (1987-1988) data.

School composition variables included school size, location, and mean SES. School size (school cohort size) ranged from 15 to 862. School location had three categories: urban, suburban, and rural. Dummy coding of school location created two variables: suburban (vs. urban) and rural (vs. urban). Therefore, urban was the baseline category against which suburban and rural were compared. School mean SES was an average measure of the SES of students in the school.

Statistical Procedure

In this study gender differences were determined for initial (grade 7) status and for rate of growth in each of the three domains of mathematical skills described above. The analyses were based on the three-level hierarchical linear model (HLM) (Bryk & Raudenbush, 1992; Raudenbush & Bryk, 1988). Six HLM models were estimated, one for each gender in each skill domain. In each HLM model the first level of the model was a set of separate linear regression models, one for each student. These regression equations modeled students' outcome scores on their grade levels:

$$Y_{ijt} = \pi_{0ij} + \pi_{1ij}(\text{grade})_{ijt} + R_{ijt},$$

where Y_{ijt} is the mathematics achievement score for student i in school j at occasion t, $(\text{grade})_{ijt}$ is the grade that student i in school j was in at the testing time t, and R_{ijt} is an error term, assumed to be independent and normally distributed with common variance of σ^2. The parameters π_{0ij} and π_{1ij} represent, respectively, the "true" initial (grade 7) status and the "true" rate of growth in mathematics for student i in school j.

The second level of the model contained two between-student regression equations modeling initial status and rate of growth based on student background covariates (variables):

$$\pi_{0ij} = \beta_{00j} + \beta_{10j}(\text{age})_{ij} + \beta_{20j}(\text{SES})_{ij} + \beta_{30j}(\text{number of parent})_{ij} + \beta_{40j}(\text{number of siblings})_{ij} + u_{0ij}$$

$$\pi_{1ij} = \beta_{01j} + \beta_{11j}(\text{age})_{ij} + \beta_{21j}(\text{SES})_{ij} + \beta_{31j}(\text{number of parent})_{ij} + \beta_{41j}(\text{number of siblings})_{ij} + u_{1ij},$$

where the βs are the parameters for the student-level covariates, and u_{0ij} and u_{1ij} are student-level error terms. Note that the student-level covariates were centered in this study so that β_{00j} and β_{01j} represented the initial status and the rate of growth in mathematics respectively for what is often referred to as a "typical" student (with the national average age, SES, and number of siblings, and who attends a school that has the same proportion of single parents as in the national sample of students). The remaining βs represent the relationships of initial status and rate of growth in mathematics to the student-level covariates.

The third level of the model included two between-school equations that regressed the average initial status and the average rate of growth in mathematics on several school-level covariates:

$$\beta_{00j} = \phi_{000} + \phi_{001}(\text{school size})_j + \phi_{002}(\text{suburban/urban})_j + \phi_{003}(\text{rural/urban})_j + \phi_{004}(\text{school mean SES})_j + v_{00j}$$

$$\beta_{01j} = \phi_{010} + \phi_{011}(\text{school size})_j + \phi_{012}(\text{suburban/urban})_j + \phi_{013}(\text{rural/urban})_j + \phi_{014}(\text{school mean SES})_j + v_{01j}.$$

In these equations, the average initial status and the average rate of growth were represented as an average value (ϕ_{000} or ϕ_{010}), an error term (v_{00j} or v_{01j}), and the contribution of each school-level covariate.

The statistical analyses were completed in two steps. First, a simple growth model was estimated without between-student and between-school covariates.[1] In this simple model, the initial status and the rate of growth in mathematics were described as an average value (fixed effect) plus a variation (random effect). This provided an opportunity to examine not only the average values of initial status and rate of growth in mathematics, but also their variances and covariances. Note that HLM obtains the estimates of the "true" variance of initial status and rate of growth as well as the "true" covariance between initial status and rate of growth. This means that the resultant estimates have been adjusted for measurement and sampling error.

The second step of analysis introduced between-student and between-school covariates, establishing a complex growth model. The purpose was to use those covariates to explain variation between students in schools and between schools regarding initial status and rate of growth in mathematics. Both the simple and the complex growth models were estimated separately for males and females across the three domains of mathematical skills: basic skills and knowledge, routine problem-solving, and complex problem-solving.

Results

The means and standard deviations for both males and females across the three domains of mathematical skills from grades 7 to 11 are reported in Table 1. As shown, both males and females grew in their mathematics achievement (across all three domains) from grades 7 to 11. Note that the range of the achievement scale was from 1 to 100 points. In basic skills and knowledge, mean gender differences ranged from 0.78 to 1.50 points, and differences in the standard deviation between males and females ranged from 0.55 to 2.47 points. In routine problem-solving, mean gender differences ranged between 1.39 and 2.00 points, and differences in the standard deviation between males and females ranged between 0.56 and 2.02 points. In complex problem-solving, mean gender differences ranged from 0.17 to 1.51 points, and differences in the standard deviation between males and females ranged from 0.62 to 1.71 points. The magnitudes of these differences are not significant in practice.

The results for the simple growth model are presented in Table 2. The top part of the table includes the parameter estimates and their standard errors for the within-student model for each domain. Statistical estimates between males and females were compared through their 95% confidence intervals with no overlap indicating statistical significance (Glass & Hopkins, 1984; Tukey, 1977).

Table 1
Means and Standard Deviations of Mathematical Skills by Gender and Grade

	Grade 7		Grade 8		Grade 9		Grade 10		Grade 11	
	Mean	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
Basic skills/knowledge										
Females	50.83	9.95	52.61	9.53	60.35	12.25	65.32	14.66	69.08	16.43
Males	50.05	10.57	51.74	10.08	58.94	13.57	64.52	16.16	68.14	18.90
Routine problem-solving										
Females	51.41	9.82	52.15	9.57	55.57	10.42	58.75	11.43	61.55	12.47
Males	49.41	10.38	50.48	11.22	54.08	11.97	57.36	13.45	60.04	14.49
Complex problem-solving										
Females	50.54	9.93	52.84	8.45	55.28	10.34	59.32	12.12	61.94	13.87
Males	50.20	10.55	51.33	9.71	55.06	11.32	58.89	13.77	61.77	15.58

Table 2
HLM Models Describing Variation in Mathematical Skills Between Students and Schools in Grade 7 Status and Rate of Growth

	Basic skills/knowledge				*Routine problem-solving*				*Complex problem-solving*			
	Females		*Males*		*Females*		*Males*		*Females*		*Males*	
Estimated parameters												
Fixed effects	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*
Average within-student model												
Grade 7 status	59.73**	(0.68)	57.94**	(0.75)	55.93**	(0.61)	53.65**	(0.66)	56.03**	(0.59)	54.79**	(0.63)
Rate of growth	4.67**	(0.19)	4.31**	(0.23)	2.30**	(0.13)	2.22**	(0.19)	2.57**	(0.13)	2.49**	(0.18)
Estimated variance components (between-student model)												
Random effects	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*
Observed parameter variance												
Grade 7 status	108.74		128.73		76.78		103.95		83.66		101.26	
Rate of growth	12.37		15.80		7.22		9.39		8.02		9.88	
True parameter variance												
Grade 7 status	87.65**	3,988.0	101.05**	3,895.9	61.96**	3,927.6	83.16**	4,229.2	68.60**	4,374.8	80.71**	4,442.1
Rate of growth	4.47**	1,674.9	5.75**	1,811.3	1.85**	1,532.3	2.34**	1,501.1	2.61**	1,646.6	2.78**	1,552.2
Reliability of estimates												
Grade 7 status	0.81		0.79		0.81		0.80		0.82		0.80	
Rate of growth	0.36		0.36		0.26		0.25		0.33		0.28	
Correlation between grade 7 status and rate of growth												
	0.95		0.99		0.64		0.74		0.82		0.89	

Table 2 (continued)

	Basic skills/knowledge				*Routine problem-solving*				*Complex problem-solving*			
	Females		*Males*		*Females*		*Males*		*Females*		*Males*	
Estimated variance components (between-school model)												
Random effects	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*	*Estimate*	*(χ^2)*
Observed parameter variance												
Grade 7 status	23.11		29.10		19.10		22.43		18.20		20.85	
Rate of growth	1.75		2.64		0.87		1.77		0.83		1.63	
True parameter variance												
Grade 7 status	18.79**	216.5	23.08**	257.8	15.43**	247.4	17.61**	257.0	14.23**	222.8	16.16**	248.7
Rate of growth	1.14**	152.3	1.86**	185.6	0.51**	131.8	1.29**	193.7	0.44**	113.5	1.13**	173.5
Reliability of estimates												
Grade 7 status	0.78		0.79		0.80		0.79		0.78		0.78	
Rate of growth	0.65		0.70		0.59		0.73		0.53		0.70	
Correlation between grade 7 status and rate of growth												
	0.67		0.63		0.31		0.32		0.39		0.38	

Note. **p<0.01. The degree of freedom for the tests are 1088 for females and 1138 for males in the between-student models. The degree of freedom for the tests are 50 for both females and males in the between-school models.

The results showed that there were no statistically significant gender differences in either the grade 7 status or the rate of growth across all three domains.

The estimates of observed and true parameter variance in the between-student model are provided in the middle part of Table 2. The estimates of true parameter variance for grade 7 status were considered fairly reliable in that the reliability coefficient, the ratio of true to observed parameter variance (Raudenbush & Bryk, 1986), ranged from 0.79 to 0.82. The reliability coefficients for the estimates of true parameter variance for rate of growth were considerably lower, ranging from 0.25 to 0.36. Note, however, that "low reliability does not necessarily mean lack of precision" (Rogosa, Brandt, & Zimowski, 1982, p. 744). It is possible to have both precise estimates of students' rate of growth and low reliability coefficients of growth scores if students' growth is not reasonably distinguishable (Rogosa & Willett, 1983, 1984; Willett, 1988).

Statistical significance of the differences in true variances between males and females across the three domains of mathematical skills was tested using Hartley's Fmax statistic (Glass & Hopkins, 1984). The results revealed that females were as homogeneous in their rate of growth as males across all three domains. In other words, as students progressed through secondary school, males and females spread out to the same degree in all three domains. However, for both males and females the variances were significantly larger in basic skills and knowledge than in the two problem-solving domains. Both males and females were equally homogeneous in routine and complex problem-solving.

The correlation coefficient between grade 7 status and rate of growth is an important indicator of the nature of the growth. All correlations were positive, suggesting fan-spread growth patterns (Bryk, 1980). That is, students with a low grade 7 status grew at a slower rate than students with a high grade 7 status. Although the extent of the fan-spread was most pronounced in basic skills and knowledge (0.95 for females and 0.99 for males), it was also apparent in complex problem-solving (0.82 for females and 0.89 for males) and routine problem-solving (0.64 for females and 0.74 for males).

The bottom part of Table 2 contains the estimates of parameter variance in the between-school model. The estimate of true parameter variance for the average grade 7 status among schools was fairly reliable (the reliability coefficients ranged from 0.78 to 0.81). The estimate of true parameter variance for the average rate of growth among schools was also reasonable with the reliability coefficients ranging from 0.53 to 0.73. Schools were considerably more homogeneous in the female average rate of growth than in the male average rate of growth in each domain. Further, for the female average rate of growth schools varied more in basic skills and knowledge than in the two domains of problem-solving. For the male average rate of growth, schools varied equivalently in all three domains.

All correlations between average grade 7 status and average rate of growth at the school level were positive, indicating fan-spread growth patterns. This means that schools with a low average grade 7 status showed a slower average rate of growth than schools with a high average grade 7 status. The extent of the fan-spread among schools was particularly pronounced in basic skills and knowledge (0.67 for females and 0.63 for males). In contrast, the extent of the fan-spread was less pronounced in complex problem-solving (0.31 for females and 0.32 for males) and routine problem-solving (0.39 for females and 0.38 for males).

The parameter estimates and their standard errors for the complex growth model that demonstrated the relationships of grade 7 status and rate of growth to student-level and school-level characteristics are reported in Table 3. The top two rows of Table 3 show the adjusted estimates of grade 7 status and rate of growth, which are the predicted values for what is referred to as the typical student (as discussed above). The typical female was almost equal in scores to the typical male in grade 7 status across all three domains. The rate of growth of the typical female was significantly lower than the rate of growth of the typical male across all three domains.

Age and SES were significantly related to grade 7 status across all three domains. Specifically, younger students (both females and males) outperformed older students, and students with high SES outperformed those with low SES. Number of parents seemed to be more influential for males than for females. Males from both-parent households outperformed males from single-parent households across all three domains. Number of siblings was not significant for either females or males across all three domains.

Age was also significantly related to rate of growth across all three domains with the exception of females in routine problem-solving. Specifically, younger students (at the same grade level) grew at a significantly faster rate than older students across all three domains (except for females in routine problem-solving). SES seemed to be more influential for females than males. Females with high SES grew at a significantly faster rate than females with low SES. For males SES had no effect on routine and complex problem-solving, but males with high SES grew significantly faster in basic skills and knowledge than males with low SES. With one exception, number of parents and number of siblings had no effect on rate of growth for either females or males across all three domains (for females number of parents positively influenced complex problem-solving).

School size, school mean SES, and school location were used as school composition covariates to account for variation in average grade 7 status and average rate of growth among schools. For both males and females school size and school location were not significantly related to average grade 7 status among schools. However, school mean SES was a significant predictor of average grade 7 status for both males and females. Schools with students from high SES outperformed schools with students from low SES. School location was significantly important for average rate of growth, but school size and school mean SES were not significantly related to average rate of growth among schools. Specifically, covariates of suburban and rural were statistically significant for females for basic skills and knowledge and routine problem-solving; females in schools located in suburban and rural areas grew at a significantly faster rate than females in urban schools in these two domains. School location, however, had no effect on male rate of growth across all three domains. Males grew at more or less the same rate whether they enrolled in urban, suburban, or rural schools.

Table 3

HLM Results Explaining Variation in Mathematical Skills between Students and Schools in Grade 7 Status and Rate of Growth

	Basic skills/knowledge				*Routine problem-solving*				*Complex problem-solving*			
	Females		*Males*		*Females*		*Males*		*Females*		*Males*	
Estimated parameters												
Fixed effects	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*
Average within-student model												
Grade 7 status	59.79**	(1.35)	59.89**	(1.39)	55.96**	(1.34)	55.78**	(1.25)	55.74**	(1.17)	56.74**	(1.18)
Rate of growth	3.75**	(0.49)	4.19**	(0.64)	1.65**	(0.37)	2.42**	(0.53)	2.03**	(0.35)	2.34**	(0.49)
Effects of between-student covariates on grade 7 status												
Age	−3.30**	(0.56)	−4.07**	(0.48)	−3.30**	(0.47)	−3.88**	(0.43)	−2.96**	(0.49)	−3.88**	(0.43)
Socioeconomic status	2.62**	(0.36)	3.06**	(0.35)	1.97**	(0.30)	2.39**	(0.31)	2.13**	(0.31)	2.41**	(0.31)
Number of parents	1.74*	(0.71)	2.29**	(0.78)	0.98	(0.60)	1.90*	(0.70)	1.31*	(0.62)	2.18**	(0.69)
Number of siblings	0.09	(0.27)	0.37	(0.26)	−0.01	(0.22)	0.34	(0.23)	0.03	(0.23)	0.35	(0.23)
Effects of between-student covariates on rate of growth												
Age	−0.64**	(0.21)	−0.80**	(0.20)	−0.24	(0.16)	−0.47**	(0.16)	−0.43*	(0.17)	−0.72**	(0.16)
Socioeconomic status	0.56**	(0.13)	0.45**	(0.13)	0.22*	(0.10)	0.06	(0.11)	0.41**	(0.10)	0.15	(0.11)
Number of parents	0.50	(0.26)	0.45	(0.31)	0.10	(0.20)	−0.17	(0.24)	0.47*	(0.21)	0.04	(0.25)
Number of siblings	0.13	(0.10)	0.19	(0.10)	0.07	(0.08)	0.08	(0.08)	0.08	(0.08)	0.09	(0.08)

Table 3 (continued)

	Basic skills/knowledge				*Routine problem-solving*				*Complex problem-solving*			
	Females		*Males*		*Females*		*Males*		*Females*		*Males*	
Estimated parameters *Fixed effects*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*	*Effect*	*(SE)*
Effects of between-school covariates on grade 7 status												
School size	–0.01	(0.00)	–0.00	(0.00)	–0.01	(0.00)	–0.01	(0.00)	–0.01	(0.00)	–0.01	(0.00)
School mean socio-economic status	4.75*	(1.72)	6.56**	(1.65)	3.86*	(1.67)	5.71**	(1.47)	4.42**	(1.49)	5.71**	(1.39)
Suburban	1.38	(1.28)	–1.43	(1.32)	1.12	(1.27)	–1.34	(1.20)	1.22	(1.12)	–1.33	(1.13)
Rural	1.09	(1.30)	–0.68	(1.32)	1.40	(1.29)	–0.34	(1.20)	1.72	(1.13)	–0.25	(1.12)
Effects of between-school covariates on rate of growth												
School size	–0.00	(0.00)	–0.00	(0.00)	–0.00	(0.00)	–0.00	(0.00)	–0.00	(0.00)	–0.00	(0.00)
School mean socio-economic status	–0.33	(0.65)	0.42	(0.76)	–0.53	(0.49)	0.09	(0.62)	–0.76	(0.47)	–0.04	(0.58)
Suburban	1.21*	(0.47)	0.34	(0.61)	0.87*	(0.35)	0.45	(0.50)	0.85*	(0.34)	0.90	(0.47)
Rural	1.17*	(0.47)	0.15	(0.61)	0.75*	(0.36)	0.21	(0.50)	0.62	(0.34)	0.31	(0.47)
Percentage of parameter variance explained (between-student model)												
Grade 7 status	11.3		17.6		11.5		15.9		10.4		16.6	
Rate of growth	10.9		7.0		3.5		1.8		12.7		8.2	
Percentage of parameter variance explained (between-school model)												
Grade 7 status	68.2		74.7		54.1		72.6		68.5		75.8	
Rate of growth	42.7		18.6		16.3		16.8		21.5		25.4	

Note. *$p<0.05$. **$p<0.01$.

The bottom part of Table 3 shows the proportion of parameter variance explained by the covariates in the model (Raudenbush & Bryk, 1988). Estimates suggested that the complex growth model was fairly effective in explaining differences in both average grade 7 status and average rate of growth among schools across the three domains (for both males and females). At the between-school level the proportion of parameter variance accounted for ranged from 54% to 76% in average grade 7 status and from 16% to 43% in average rate of growth. These proportions are often considered meaningful in behavioral sciences (Cohen & Cohen, 1983; Rosenthal & Rubin, 1982). At the between-student level the proportion of parameter variance accounted for (across gender) ranged from 10% to 18% in grade 7 status and from 4% to 13% in rate of growth.

Discussion

The present study is unique in three ways. First, it emphasized students' rate of growth in mathematical skills (the dynamic aspect of mathematics learning) rather than their level of skills in one particular grade (the static aspect of mathematics learning). Second, gender differences in the acquisition of mathematical skills were examined through a nationally representative sample. Finally, it used HLM techniques to examine variation in rate of growth among students and among schools and the strength of the relationship between rate of growth and student background and school composition characteristics.

The two principal findings of this study are that males and females grew at the same rate across all three domains (basic skills and knowledge, routine problem-solving, and complex problem-solving) and that students' growth in these domains was consistently fan-spread as they moved from grade 7 to grade 11. This means that the variation in mathematics scores increased as students progressed through secondary school, and students maintained their initial position in the distribution. These results are similar to those reported by Willms and Jacobsen (1990).

Therefore, there is more evidence now that students become increasingly diverse in their mathematics ability as they progress through secondary school. This situation creates a serious challenge for mathematics teachers, particularly in senior high schools. However, there are ways to avoid or reduce this problem. First, mathematics teachers may consider the idea of "content differentiation" as a way to diversify mathematical knowledge levels to suit the differential needs of students (National Council of Teachers of Mathematics, 1989). Second, the high correlation between grade 7 status and rate of growth suggests that mathematics teachers may use grade 7 status to locate students who are likely to have difficulties in mathematics during the subsequent grades. Early intervention and special attention may then be effectively directed to those students. Finally, remedial programs may be considered in grade 7 to overcome students' previous difficulties in elementary mathematics. Overall, mathematics teachers may wish to make certain that their students have a solid mastery of grade 7 mathematics.

Another important finding of this study is that the extent of the fan-spread was more pronounced for basic skills and knowledge than for both routine and complex problem-solving. Thus it is ability to apply basic skills and knowledge more than ability to solve routine and complex problems that separates students in mathematics. This finding has three practical implications. First, rather than overemphasizing problem-solving, mathematics teachers may wish to devote enough instructional time (particularly in grade 7) to basic skills and knowledge. Second, the teaching of problem-solving may incorporate effective reviews of basic skills and knowledge. Third, basic skills and knowledge should be monitored frequently, and remedial measures should be available.

Willms and Jacobsen (1990) reported that the extent of fan-spread was greater for males than for females. However, in the present study females were found to be as variable as males. This implies that gender differences in fan-spread are more likely to be a local phenomenon rather than a national one (Willms & Jacobsen used a sample from one Canadian school district). This finding does not fit well with the literature in which gender differences appear to be more a global rather than a local phenomenon (Ma, 1995). Given that the existing literature on gender differences mainly examines students' status rather than rate of growth in mathematics, this study extends the literature in a meaningful way—gender differences in students' status in mathematics may be a global phenomenon, whereas gender differences in students' rate of growth in mathematics may be a local phenomenon.

Among student background characteristics examined in this study, age was the most significant predictor of rate of growth in mathematical skills (for both males and females), and SES was also a significant predictor for female rate of growth. Older female students from disadvantaged socioeconomic background thus need more help in their study of mathematics. Number of parents and number of siblings had no effects on rate of growth for either females or males across all three domains. This suggests that students from single-parent families or from large families are not necessarily disadvantaged in the development of mathematical skills.

The most important finding at the school level is that the variation among schools in their average mathematics achievement increased over time, and schools maintained their initial position in the distribution. The average grade 7 status of schools can, therefore, be used to identify roughly those schools that may need attention and intervention in the teaching of mathematics during later secondary grades. The extent of fan-spread among schools was more pronounced in basic skills and knowledge than in both routine and complex problem-solving. Therefore, it is basic skills and knowledge rather than problem-solving that separates schools in mathematics. Schools that successfully emphasize basic skills and knowledge appear to be more effective in teaching mathematics.

School composition (school size, school mean SES, and school location) had no effect on male rate of growth across all three domains. What determined the later status of a male student in mathematics was the average grade 7 status of the school in which he enrolled. This implies that attending top-performing schools is particularly important for males to excel in mathematics. Females in schools located in suburban and rural areas grew at a significantly faster rate than females in urban schools. This indicates that a top-performing suburban or rural school provides a far better chance for females to do well in mathematics. One possible reason for these location effects is that females are more sensitive to school environment characteristics such as disciplinary climate.

Individual change or growth has been a research focus in behavioral sciences for a long time, yet "many concerns catalogued by Harris (1963) still continue to trouble quantitative studies of growth" (Bryk & Raudenbush, 1992, p. 130). Research on individual growth has been plagued by difficulties in conceptualization, measurement, and design.

> The development of hierarchical linear models, however, now offers a powerful set of techniques for research on individual change. When applied with valid measurements from a multiple-time-point design, they afford an integrated approach for studying the structure and predictors of individual growth. (p. 131)

The present study demonstrates the usefulness of HLM techniques in future studies of individual change.

The variables included in this study were restricted to variables descriptive of school composition. Future studies should include variables describing schooling processes such as differential retention rates, various course-taking patterns in mathematics, and characteristics of the mathematics curriculum in an attempt to understand better the rate of growth in mathematics by males and females. Researchers might also consider traditional components of schooling processes such as disciplinary climate and school leadership as they relate to the rate of growth in mathematical skills. Results from these studies will help identify characteristics of schools from different perspectives of school life that promote better teaching and, subsequently, learning in mathematics by both males and females.

Note

1. The simple growth model contained three levels. The first level of the model (within-student level) was:

$$Y_{ijt} = \pi_{0ij} + \pi_{1ij(grade)ijt} + R_{ijt},$$

where Y_{ijt} is the mathematics achievement score for student *i* in school *j* at occasion *t*, $(grade)_{ijt}$ is the grade that student *i* in school *j* was in before testing occasion *t*, and R_{ijt} is an error term. The second level of the model (between-student level) was:

$$\pi_{0ij} = \beta_{00j} + u0ij$$
$$\pi_{1ij} = \beta_{01j} + u1ij,$$

where the βs are intercepts, and u_{0ij} and u_{1ij} are error terms. The third level of the model (between-school level) was:

$$\beta_{00j} = \phi_{000} + v00j$$
$$\beta_{01j} = \phi_{010} + v01j.$$

The average initial status and the average rate of growth were represented as an average value (ϕ_{000} or ϕ_{010}), and an error term ($v00j$ or v_{01j}).

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