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Jérôme Proulx
University of Ottawa

Between Meaning-Making and Learning the "Rule": The Case of a Prospective Teacher of Mathematics at the Secondary Level[1]

This article reports on a case study intended to improve understanding of the development and characteristics of prospective mathematics teachers' oral explanations. The teaching practices of Donna, who taught algebraic operations, were analyzed, after which she was interviewed. The presence of an important tension in how Donna explained mathematics emerged where her explanations were divided between contextualized understandings and formalized knowledge, highlighting the difficulties in teaching of transition from one to the other. The article concludes with a discussion of a plausible rationale for this tension, which emerges from an understanding of mathematics as reduced to a call for formalized procedures.

Cet article présente une étude de cas visant l'amélioration des connaissances sur le développement et les caractéristiques des explications orales que fournissent les enseignants candidats en mathématiques. L'auteur a analysé les pratiques d'enseignement de Donna, qui a présenté des opérations d'algèbre, et a ensuite passé la stagiaire en entrevue. La présence d'une tension importante dans les explications mathématiques de Donna s'est fait sentir quand ses explications étaient partagées entre des connaissances contextualisées et des connaissances formelles, mettant en évidence la difficulté de passer d'une sorte de connaissances à l'autre. L'article conclut avec une discussion sur une explication plausible de cette tension en tant que résultat d'une interprétation des mathématiques comme n'étant que des procédures formalisées.

Context of Research

Many studies show the importance of mathematics teachers' oral explanations for the development of students' understandings and reasonings and for the establishment of links between notions (Ball, 1991; Hersant, 2001; Mopondi, 1995; Nolder, 1991). These explanations also appear to be of central importance in the establishment of a mathematical culture in the classroom, promoted by how teachers manage errors and make use of metaphors in their classrooms (Bauersfeld, 1998), bring forth the mathematical arguments (Cobb & Yackel, 1998) and the format used in presenting them (Krummheuer, 1992), and negotiate the mathematical understanding in the classroom with students (Voigt, 1994). However, it seems that for a number of prospective teachers, "speaking the mathematics" in class or simply explaining with words the mathematical understandings is not an obvious activity (Ball, 1988). Preservice teachers have difficulties using metaphors and everyday language when they explain mathematics, and therefore have a tendency to impose a technical and restrictive language of mathematics.

Jérôme Proulx is an assistant professor of mathematics education. His research interests are situated in teacher education, particularly the mathematical preparation of mathematics teachers.

> For many teachers, the strength and the generalizability of mathematics is inseparable from the strictness and precision of its underlying representations, be they verbal or otherwise symbol using. Like priests who celebrate the esoteric language game of their caste, many mathematics teachers constantly insist on stating, and having stated to them, subject matter in "true" mathematical language—that is, on which is as "fine-tuned" as possible. (Bauersfeld, 1998, p. 223)

This situation prompted my interest in knowing more about how this competence (i.e., "speaking the mathematics") is being developed by prospective teachers. It is with this in mind that I have become interested in oral explanations developed in action by prospective teachers of mathematics at the secondary level, in an attempt to better understand and better situate the possible characteristics of these oral explanations.

Bear in mind, however, that trying to describe and characterize the nature of oral explanations cannot be realized without taking into consideration the background influences underpinning these very oral explanations: that is, the teaching intentions and influences that are orienting the prospective teacher to opt for a certain type of oral explanation instead of another. These two aspects (description of the characteristics and the rationales underpinning them) of oral explanations have the potential to enable a better understanding of the process of constructing/learning this professional competence by student teachers, and of bringing forth some issues in this process. Therefore, two specific questions framed my research.

1. What are characteristics of prospective secondary-level mathematics teachers' oral explanations when they are teaching (in their practicum)?
2. What are the possible influences that are orienting these oral explanations?

To address these research questions, a multicase study with five preservice teachers of mathematics at the secondary level was conducted (Proulx, 2003). Here I report on one of these cases in specific details: Donna (pseudonym) who taught the introduction to algebraic operations in grade 9. I have chosen to address in detail this specific student teacher because of a particular and significant issue in how she orally explains mathematics: the presence of an explicit and felt tension (disconnection) in her attempts to make the transition from a more concrete to a formal/abstract level in her mathematical explanations.[2]

Theoretical Clarifications

Mathematical discourse can be characterized as any attempt to communicate, interact, reflect, and render explicit content related to mathematics. This discourse can be seen under two main aspects: public discourse and private discourse. Public discourse can be described as an interaction or the implied interaction of a minimum of two persons (i.e., a speaker and a listener). As for private discourse, it is mostly personal reflections and internal mathematical thoughts (Sfard, 2001). Obviously these different aspects of discourse, however, share the same intents: to clarify, explore, explicate, render explicit, reason, and make understandable the concepts under study. This study, by being situated at the level of future mathematics teachers and their classroom, focuses on the first aspect: the public mathematical discourse. This public mathematical discourse can also take many forms: gestures, words, images, writings, and so forth. Here the research attempts to describe the public mathematical discourse at the level of the use of words, sentences, and phrases used to communicate mathematics and to render explicit a notion, a particular reasoning, a mathematical phenomenon, or a mathematical activity to the students.

Elements of Methodology

In this section I focus mainly on aspects of the methodology (used in the larger study) that are relevant to Donna's case.[3] All the preservice teachers were in the second year of their mathematics teacher education program in a large university in Quebec. They were completing a second practicum in mathematics teaching.

The data were gathered from two specific sources. First, each student teacher gave three 75-minute videotaped classroom lessons taken at various points during their six-week practicum, with the camera placed at the back of the classroom, capturing the teacher's and the students' explanations and interactions. This enabled me to provide a characterization and a description of the oral explanations used in their classrooms. The reason for obtaining three videotaped lessons was that it provided me with a considerable range concerning their teaching at various moments, and therefore offered a better perspective on their oral explanations. Second, I conducted an individual semi-structured interview with each student teacher after the practicum to enable me to underline various rationales underpinning his or her oral explanations given in class.[4]

Analysis of Oral Explanations Through the Videotapes

To analyze the videotapes, a frame of analysis was constructed that comprised 10 specific dimensions (see Table 1).

For the purpose of this article, I specifically address four of these dimensions, as they illustrate well the issues of tension present in Donna's oral explanations: *the nature of the oral explanations, the openness to different answers and student strategies, the flexibility in the oral explanations, and the type of language used*. (For a detailed account of the aspects of the frame of analysis and how it was systematically used in all five cases for the 10 dimensions, see Proulx, Bednarz, & Kieran, 2006.) In the dimension *nature of the oral explanations*, I paid attention to whether the oral explanations were centered on techniques and knowhow, on the development of understanding, reasoning, and meaning-making, on memorization, on the explicit detail of procedures to follow, on formulas or on instrumental and relational understanding (Skemp, 1978). In the dimension *openness to different answers and student strategies*, I looked to see if the oral explanations were open to diverse strategies, if they were leading to a unique answer and a unique way of doing, if they were focusing on the reasons underpinning a strategy, and so forth. In the dimension *flexibility in the oral explanations*, I analyzed the oral explanations to see if they were mostly centered on a repetition of the same explanation (many times), on a use of varied examples and varied levels of language, on a "revoicing" (Forman & Ansell, 2001) of the student's explanations, and so forth. Finally, in the dimension *the type of language used*, I focused on the use of metaphors and analogies, of everyday language, of a technical language and/or a precise vocabulary in regard to mathematics, and so forth.

Table 1
Analytical Frame for Mathematical Oral Explanations

10 dimensions of mathematical oral explanations
The role and the place of students in the oral explanations given
The sort of questioning
The nature of the oral explanations
The openness to different answers and student strategies
The creation of links between concepts
The type of language used
The flexibility in the oral explanations
The presence of mathematical verbalizations
The status of the oral explanations in the teaching
The mathematical validity of the oral explanations

The data from the videotaped lessons were broken into units of analysis that consisted of one explanation given on one specific element by the prospective teacher. Therefore, it was possible that an interaction with one or more students took place in the same explanation about the same issue, which consisted of one unit of analysis. A new unit of analysis was created when a new explanation was given about another element.

Constructing and Conducting Individual Interviews
Each semistructured interview protocol was constructed on the basis of the analysis of the three videotapes. The interview protocol contained two parts: an individualized section focusing on specific aspects tailored to the events of each individual teacher's videotapes and a common section addressing general notions about mathematics teaching. The questions for the individual part were concerned with and organized around the teaching practices observed on the videotapes. Hence the questions were rooted in the practices of each prospective teacher.

In the case of Donna, some questions were directed at her classroom routines and at specific events in which she used particular strategies (a variety of explanations, metaphors, analogies, links between notions, drawings, visual support, etc.). In her lessons Donna chose always to use the same specific sequence to introduce a new algebraic operation to work on: (a) mental arithmetic calculations reflecting the algebraic operations to work on (i.e., the distributive law), (b) rectangular-area calculations again reflecting the algebraic operations to work on, and (c) taking note of the mathematical abstracted rule to solve the algebraic operation (i.e., the first monomial in the binomial is multiplied with the first monomial in the second binomial). Therefore, I was interested in knowing more about her reasons for choosing this specific recurrent routine in her teaching. Here is an example of a question that I asked Donna about her classroom routines.

> *I noticed that, in your three lessons, you were always using the same approach: mental arithmetic, rectangular-area calculations, and followed by the taking of notes, by the student, of the rule to use for solving.*
> *(a) Why did you always function like this? What were your reasons?*

(b) Where did this idea of acting in that way come from? What made you think of all this?
(c) Could you have changed the order in your presentation? Why?

The second part of the interview, identical for all the preservice teachers, comprised questions at a more general level. I invited participants to talk about their past as secondary-level students, their experiences as student teachers, the possible influence of their practicum supervisor, the students, the textbook, and so forth. Here are questions that were asked about future teachers' past experiences as students at the secondary level.

Can you explain how you were taught the [algebraic operations] when you were a student at the secondary level?
Did the education that you received as a secondary level student played a role in the way that you introduced and taught [algebraic operations]?
If yes, how and why? If not, why?
Do you have particular or specific examples where it helped you?

The intention behind the interviews was to gain a sense of the rationales underpinning the prospective teachers' oral explanations. Therefore, in the analysis I looked for specific themes that were raised by each student teacher that characterized the origins and reasons behind his or her oral explanations. In the case of Donna and the particular phenomenon that I address in this article (i.e., the tension apparent in her oral explanations), I use her interview utterances to support and give sense to the descriptions and characterizations of her oral explanations that I observed and extracted from the videotapes.

I now turn to an analysis of the data gathered from the videotapes of Donna's lessons and the following interview with her.

Analysis of Donna's Case

Before entering in the analysis, an explanation of the general structure of Donna's typical lesson is needed. Her lessons are divided into three parts.[5] The first focuses on a revision of the homework or the mini-test given in the previous class; the second is centered on mental arithmetic with questions using arithmetic operations that are linked to the algebraic operations to study (i.e., the distributive law) and on area calculation contexts using the same arithmetic or algebraic operations; and the third part, used as a conclusion for the class, focuses on the rule[6] or the technique for carrying out the algebraic operations (i.e., the multiplication of binomials). With this in mind, I turn to four specific dimensions in her oral explanations (i.e., the nature of the oral explanations, the openness to varied student answers and student strategies, the flexibility in the oral explanations, and the type of language used).

Characteristics of Donna's Oral Explanations

The nature of the oral explanations

Donna's oral explanations can be divided into two distinct types. The first is centered on the mathematical reasoning underpinning the concepts being studied and the construction of meaning by the students. This type of explanation is used in the second part of the lesson: that is, when she works with mental arithmetic and rectangular-area calculations to bring forth a meaning to algebraic manipulations. The second type of oral explanation is centered on the explanation of the rule and the procedure to follow; it is used in the first (homework and mini-text revisions) and third parts of the lesson (conclusion).

When she uses this second type of explanation (in relation to the algebraic operations to solve), she completely changes her way of explaining: the explanations mainly concern the functioning of the rule and are no longer centered on the inherent mathematical reasoning—no links are made with the prior activities of mental arithmetic and rectangular-area calculations to make sense of the algebraic operation/manipulations. In other words, from the moment that she is no longer working with the mental arithmetic or the rectangular-area calculations she seems to concentrate exclusively on the operations and their functioning in a purely algebraic context. (The importance that Donna places on the rule was later confirmed in the interview.) When she introduces the algebraic rule to work with (i.e., monomial multiplied by monomial), she takes this rule for granted and chooses not to go back to the prior activities, knowledge, or meanings developed in mental arithmetic and rectangular-area calculations.

Thus in Donna's teaching two disconnected types of oral explanations are present: the explanations *before*-the-rule (in context and centered on mathematical reasoning and understanding) and explanations *after*-the-rule (centered on a "formal" procedure to apply, i.e., the rule). Here is an example of each type of oral explanation in relation to a rectangular-area calculation problem (see Figure 1).

[In the before-the-rule explanations, Donna presents the answer [(9–a) · x = 9x – ax] using the figure.]
Donna: Here are the two ways to procede: you first calculate the depth, so it is "9 – a." Then, you multiply by "x" to obtain the area of the rectangle. There is the other way to proceed, it is to calculate all the area of the rectangle, so "9" times "x," and then you subtract the area of the small rectangle that is "9a," I mean "x" times "a." So there are two ways to proceed and it gives the same result. So here, [she points to the following expression on the board]: (9 – a) · x = 9x – ax], we have to multiply "9" by "x" and "–a" by "x." And this gives us "9x – ax."

Then toward the end of the lesson in the *after*-the-rule part, she gives the following rule in relation to the same rectangular-area calculation problem (Figure 1).

Donna: To multiply a monomial by a polynomial, you multiply the monomial by each of the terms of the polynomial. By taking into account the signs, obviously. [She then writes the definition on the board and asks the students to take note of it.]

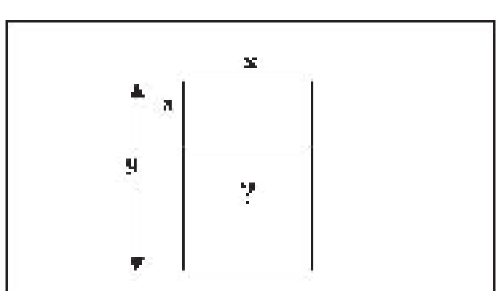

Figure 1. First rectangular-area calculation problem given by Donna (for the distributive law).

The way this rule definition is given is disconnected from the previous work on rectangular-area calculations and on the mental arithmetic, which does not produce a transition from one way of working to the other, but mostly produces a clash where no apparent links are observable from the two ways of working with the same concept. Later in the lesson, she offers the following explanation about the procedure/rule to follow concerning the multiplication of binomials (*after*-the-rule part).

Donna: We are going to do this one: (2a + 3)(a – 8) [she writes it down on the board]. You multiply "2a" by "a," it gives "2a" exponent "2." "2a" times "–8" [she points to them], it gives?
Group: [recording unclear]
Donna: "–16a." "3" times "a" [she points to them]
Group: "3a"
Donna: Plus "3" times "–8"?
Group: "–24"
Donna: "–24." So here [she traces a line under "–16a" and "3a"], I have two terms alike. I can add them: "2a" square, minus "13a," minus "24" [$2a^2 - 13a - 24$]. Perfect!

From a context linked to concrete elements (i.e., mental arithmetic and area calculations), Donna jumps to a formalized and abstract way of operating for the same concept. This creates a tension between how she explained in the previous set of activities and where she wants to lead students (i.e., the formalized rule). These two types of oral explanations characterize well the nature of Donna's oral explanations.

The Openness to Different Answers and Student Strategies
In the part that I call *before*-the-rule, Donna creates a classroom environment open to varied answers and solutions by inciting the students to explain their procedures, to offer alternative solutions, and to question the validity of the answers given by others by asking questions such as: "Is there another solution?" and "Does someone have another procedure?" Many solutions are then offered. In addition, students frequently offer judgments (almost naturally and without being asked) on the varied solutions that are given by other students in mental arithmetic and area calculations. For example, for the multiplication of 12 x 13, the students offered the following solutions for solving mentally: 12 x 12 + 12, 12 x 10 + 12 x 3 = 12 · (10 + 3), and 13 · (10 + 2). Donna took specific care of outlining all these solutions on the board and explaining them in detail by explaining the links relating each solution to the other and how it gave the same result but from other strategies.

However, in the *after*-the-rule part, her openness is not as clear. She seems mostly to use one way to solve the problem, and it is the rule that she establishes. Again, this emphasizes another way of working and operating when Donna is in the *after*-the-rule part; varied ways of solving are switched to only one way of operating to find a solution.

The Flexibility in the Oral Explanations
Donna has many approaches for explaining a notion and seems flexible in her teaching. When she converses with a student who does not understand, she adjusts her oral explanations and changes them significantly. In addition, she frequently reworks and reformulates the oral explanations of students by

repeating them, clarifying them, adapting them by adding or insisting on specific aspects: a practice that Forman and Ansell (2001) call revoicing. Here is an example of revoicing or of reformulation in relation to a second problem of rectangular-area calculation (see Figure 2).

[Following the answers given for the questions: "With what do I multiply 4 to obtain an area equal to x?"; "With what do I multiply 4 to obtain an area of 3x?," one student intervenes to verify the validity of the answers for figure 2.]
Student: "3x over 4" plus "x over 4," it gives "x."
Donna: OK, excellent. So here, I can also write it like "3x" plus "x" over "4," it gives me "4x" over "4," and I find the same thing that I found before.

Here is an example of the variety of explanations that Donna offers to a student who experiences difficulties with the fact that "x/4" multiplied by "4" gives "*x*" (see Figure 2). Donna then explains it in another way (see Figure 3).

Donna: Look, if for example I have a segment here that is "x" [She traces a segment of length "x"]. OK? I divide this segment in 4 parts. What will I obtain? Each part will measure how much?
Student: One quarter of "x."
Donna: This, will measure a quarter of "x" [She then shows one part of her segment that measures "x/4" like in the figure 3]. All this, it is "x," and this is a quarter of "x" [she points to it]. By what, how many times do I have to repeat this measure [she points to "x/4"] to obtain this one [she points the whole segment, the "x"]?
Group: 4.
Donna: 4 times. 1, 2, 3, 4 [she counts on the drawing]. Is it ok? I repeat 4 times the quarter of "x" to obtain "x."

This flexibility is, however, noticed only when she is working in the contexts of mental arithmetic and rectangular-area calculations and not in the *after*-the-rule part. As it appears in the previous dimensions of her oral explanations, she also seems to focus exclusively on the rule and loses the flexibility that she enacted and that was characteristic of her work on mental arithmetic and area calculations (which was directly linked to the same algebraic operations/manipulations).

The Type of Language Used
Donna uses everyday language in her explanations and adapts her language to that of the students by taking care of clearly verbalizing the reasonings. Also, as noticed above, Donna "speaks the mathematics" in varied ways. She sometimes explains some notions using three ways. Here is an example.

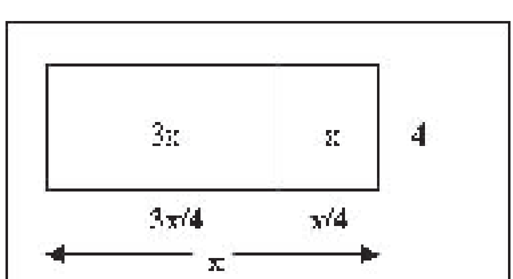

Figure 2. Second rectangular-area calculation problem given by Donna (for division).

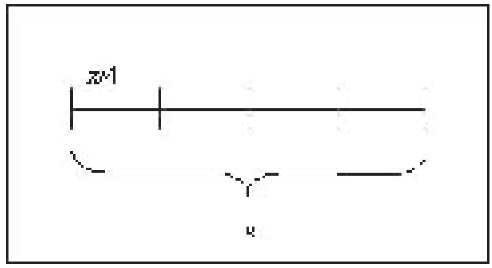

Figure 3. Figure used by Donna to explain to a student experiencing difficulties.

> *[To solve the problem: $(x + 5)^2$]*
> *Donna: So for the number 10, "(x + 5) exponent 2," it is the same thing as "(x + 5)" that I multiply by "(x + 5)." So "(x + 5) exponent 2," it means that I multiply the expression in parentheses by itself.*

However, in the *after*-the-rule part, some inflexibility (rigidity) is observable in her language when it is time to work on the rule, and also in regard to anything that comes from the domain of pure/abstract mathematics: what Bauersfeld (1998) has referred to as a sort of celebration of technical language. Here is an example:

> *Donna: OK, can you repeat what we just said ? If I want to divide a polynomial or a binomial by a constant, what do I have to do?*
> *Student: Well, you take the first digit, and you divide it by …*
> *Donna: How can I call that the first digit? This is not a digit [she points to "4x"], what is it?*
> *Student: Well, a term.*
> *Donna: A term. So the first term …*

As for the previous three dimensions, two types of explanations (or two types of language) seem to be used here. This provokes a disjunction between how she uses terms in everyday language in activities that are contextualized and uses rigid/formal mathematical language in the "rule part."

Summary of Donna's Oral Explanations

The analysis of Donna's oral explanations leads to the observation of a tension in how she speaks the mathematics: a sort of rough passage without apparent connections between the *before*-the-rule part and the *after*-the-rule part. When she works on the rule, no connections are made in relation to the previous contexts of mental arithmetic and rectangular-area calculations. And there is no trace of openness to other answers and solutions, of creation of equalities, of flexibility, and importantly, of participation of the students in the ways of solving. When she is in the *after*-the-rule part, Donna gives the rule and students take notes; but when she is in the *before*-the-rule part, the students participate actively in the discussions, and Donna works in flexible/various ways to arrive at answers and create meaning. Simply put, in the *after*-the-rule part, Donna's choice of language and actions become rigid and centered on one aspect: the rule.

From observation and analysis of her lessons, the rule seems to be the way to conclude the study of notions (here algebraic operations). It appears to represent the main objective or goal of her lesson (she confirms this in the interview). This could be linked to what Brousseau (1998) calls the *institutionalization* phase: Donna tries to establish the algebraic principles to know (the rule) to be able to solve the particular type of problem at hand. It represents her way of concluding a specific notion. This institutionalization of the rule seems important in Donna's eyes because from the moment that it is done (i.e., that the algebraic principle has been shown using the rule), she immediately takes it for granted and does not return to its underpinning meanings in the mental arithmetic and area calculations activities that she had used to build it: she focuses on the rule and on its application for solving other problems, assuming that students are ready to go from one form to the other or that they will see the transition between the different modes.

The Rationales Underpinning the Oral Explanations: The Interview[7]

Donna was well aware of this tension in her teaching and was also questioning herself about it. She explained in the interview that the same phenomenon appeared in her third practicum of teaching (that she had just completed after her second one because of schedule constraints), where she had to teach fractions and decimal numbers. Again, this phenomenon permeated all her teaching practices where a gap existed between the contextualized learning and the formal one: that is, between the meaning-making construal and the institutionalization of rules to operate on fractions and decimals. She summarized her view of this tension and her questioning of it with the following assertion: "So, humm … something is probably missing, but I don't know what for the moment. But something is missing." (All Donna's interview utterances are reported in quotation marks.)

For Donna, "the rule, it's the objective of the lesson." This brings her to show the rule and to insist on it in the conclusion of her lessons. Also, for her the rule gives meaning to the learning that happened: "The students had to see that what we did, it had a purpose, and that it leads to something that could be general." Donna considers knowledge of the rules of central importance for learning algebraic operations.

Another point that stems from the interview is the fact that the teaching she received when she was a student at the secondary level was focused on the learning of these rules: "It's true that before, in the lecturing model of teaching, they were teaching us the rule … the teacher comes, he [or she] gives the rules, then he [or she] gives some examples, and then we apply them." It is an aspect that appears to be of particular interest because from all the things that she mentions in her interview, it is the only instance showing and highlighting a possible influence linked to the use and importance of the rule (the other utterances of the interview are always focused on the meaning-making construal by students), and it seems to shed some light on why she insists so much on the rule.

In the interview, Donna explains that construing meaning and developing reasoning and understanding are very important for her. She underlines the importance of verbalizing this reasoning, of working on many solutions, and of varying the explanations to reach the largest possible number of students. In addition, she explains that her recent experiences in her mathematics teacher education program, her previous experiences of teaching adult students, and her experience as a mother play important roles in her teaching. In her mind all these experiences sensitized her to the importance of the learner's participation in her teaching: "You cannot teach if students do not have an effect on you. It is impossible because it means that if they do not have an effect on you, you are only doing your own way. It is impossible." She then asserts that it is important for her to adapt herself to the students and to their understanding, to make them work in already known fields, and particularly to start from the students' prior knowledge to develop the notions to work on:

> *So starting from an already known theorem, place the student in a comfortable zone, starting from his or her previous knowledge, his or her skills … Also, because it is really him [or her] who has developed that. And he [or she] knows how to use all that. Then, to go towards something that he [or she] does not know at all and to realize that finally it is just a question of reflecting a little … because that is learning. It is to know some things and to be able to use them anytime.*

The link to her teaching practices is palpable, especially in her tendency to work first on the mental arithmetic and the rectangular-area calculations as "known grounds" and then to expand from them and to build on students' knowledge. Hence whereas most of her experiences have brought her to lead students to construct meaning of the things they were working on, her experience as a student appears to have brought her to see the objective/end goal of mathematics as about learning the rules. She works on reasoning at first, but feels the urge to go to the other type of work (i.e., the rule) because if not, it has "no meaning."

Her past experiences as a student seem to have played an important role in her vision of the topic to teach (algebraic operations) and on her teaching practices. This is closely linked to Bauersfeld's (1998) point when he asserts that the teacher's experiences as a student are not to be neglected as orienting elements in the decisions and teaching practices enacted in class. It is also linked to Ball (1988), who suggests that if prospective teachers have succeeded in mathematics as students, they will tend to approve and reenact the same methods to which they were exposed.

Even if she is aware of the presence of a tension in her practice, Donna intends to start from the mental arithmetic and rectangular-area calculations to lead the students to the rule. She wishes the rule to flow naturally from the previous activities of mental arithmetic and area calculations because this is where it should be heading. However, she notices, as I observed from the videotapes, that it is not exactly the case in her practice; it does not flow naturally.

Conclusions on Donna

In Donna's interview as much as in her oral explanations in class, there is the presence of this tension between concrete and formal/abstract forms. Donna's intention is to work on the development of meaning and understanding for the students in a progressive and adaptive manner. The conclusions of her lessons, however, are oriented toward the rule and its institutionalization. This tension creates a significant clash in her teaching approach. This is a conflict of which she is herself fully aware and with which she is not comfortable. Yet she does not know what to do. Concluding her lessons with the rule seems to be something of extreme relevance and importance to Donna.

Because the conclusion and the objective of her lessons are always directed toward the rule, it is possible to think that the part about mathematical understanding (mental arithmetic and rectangular-area calculations) constitutes some sort of prerequisite for being able to construct a meaning of and to lead toward the teaching objective: the rule. Using analyses of teaching practices conducted by Bednarz and Giroux (in press), it is possible to think that her clear choice for the rule indicates that the two types of explanation (meaning construal vs. rule) do not have the same status or the same importance in regard to the lesson objectives or simply in regard to the topic being studied. This imbalance plays an important role in creating the tension in her oral explanations.

Making Sense of the Tension: Reflecting on Issues About the Nature of Mathematics

The presence of this tension in Donna's oral explanations represents an important problem requiring further investigation and close attention. In effect, it represents something that mathematics teachers also appear to struggle with in their everyday practices (Bednarz & Perrin-Glorian, 2003). This illustrates the need to better understand this specific issue.

The data reported here shed some significant light on this issue of tension, as Donna's divergent influences (as a student learning mathematics and as a student teacher learning how to teach mathematics) appears to be dividing her between two approaches in her teaching that she tries with difficulty to reconcile. But above all, this phenomenon points to something fundamental: the presence/importance of rules and procedures in learning mathematics. Donna's "stubborn" intention of ending with the rule and her inability to see how it could end otherwise ("The rule, it's the objective and its how you conclude the topic, I don't see no other possible options") demonstrates her attachment to or understanding of the concepts of study as represented and summed up by mathematical rules to follow, as if rules or procedures represented the entire goal or summit of the mathematical activity.

This conception of mathematics as being all about rules/procedures is, however, widespread (Battista, 1999), neglecting the fact that there is much more to mathematics than procedures and that procedures do not summarize or represent mathematics, but only a portion of it. Bourbaki (1950) and Brousseau (1988).

> What all this amounts to is that mathematics has less than ever been reduced to a purely mechanical game of isolated formulas; more than ever does intuition dominate in the genesis of discoveries. (Bourbaki, p. 228)

> It is true that at some point students will have to learn some things that were produced in the past, but it does not represent the essential part. The essential will be to work with this "knowledge" in conjunction with the meaning it can have. (Brousseau, my translation)

To take only the procedural and calculational aspects of the mathematical enterprise into consideration is to make a dangerous mistake for education in mathematics, explains Battista (1999), where mathematics becomes perceived as a discipline made of facts to memorize, recipes to follow, and mimic and drill-practice instead of being perceived as a creative human enterprise of inquiry (Schifter & Fosnot, 1993). Mathematics is filled with concepts, notions, and ideas that have structures and interrelated relationships. Doing mathematics entails deducing, relating ideas, conjecturing, analyzing phenomena, judging and testing, making inferences, recognizing and describing patterns, experimenting, building models, noticing representations of phenomena, and so forth. Hence procedures are only a part of the mathematical activity; there is much more.

It is important to understand that teachers' understanding of the mathematical content to teach has an important effect on their teaching and how it will be offered and approached in the classroom. Therefore, a teacher who conceives of algebraic operations as being ultimately about rules will teach this way in the classroom. This is something that Hersh (1986) stressed.

> One's conception of what mathematics *is* affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it.… The issue, then, is not, What is the best way to teach? But, What is mathematics really all about? (p. 13, original emphasis)

Therefore, Donna's tension lived in her practice will possibly remain as long as her conception of this topic stays the same: that is, as being ultimately about rules to apply. There is, then, much more than the unequal status or the higher value given to the purely/abstract algebraic work in comparison with the contextualized work. The issue also concerns Donna's understanding of the nature of the topic. Her conception of the central importance of knowing the rule to solve algebraic operation structures her teaching and appears to play a significant role in creating the felt tension in her oral explanations.

Notes

1. A shorter French-language version (pre-publication) of this article appeared in Proulx (2004).
2. For these reasons this report on Donna should not be misinterpreted as representative or illustrative of the other preservice teachers in the study. For an overview of all five cases, how they relate to each other and the overarching questions of the research, see Proulx (2003) or Proulx, Kieran, and Bednarz (2004).
3. Again, for more information and precise details on the entire methodological orientation and data collection process used in the larger study, see Proulx (2003) or Proulx et al. (2004).
4. The choice of the second practicum of teaching was mainly technical, as the first is for observation only, the third concerns a second topic of instruction, and the fourth ends their program and student teachers often obtain teaching contracts to complete the school year and do not come back to the university afterward. Choosing the second practicum gave more flexibility in obtaining the videotapes and conducting the interviews.
5. What I mention above about Donna's recurrent classroom routines refers exclusively to her teaching/introduction of a new algebraic operation to work on. Here I refer to the whole structure of her lessons.
6. The word *rule* was explicitly used by Donna in the interview. Thus I use the word *rule* to represent the procedure to use to complete the algebraic operations.
7. As mentioned above, I report on aspects of the interview that are relevant to the presence of the tension in her oral explanations.

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